



CALCULATION POLICY

This calculation policy has been written to provide an understanding of when and how the four operations (addition, subtraction, multiplication and division) are taught within the Busy Ant Maths scheme of work, which has been adopted by HCFS. It has been designed to ensure consistency throughout the whole school and to make teachers aware of the continuity and progression in skill development across the year groups.

The policy aims to enable staff (and parents) to see how the:

- core mathematical representations;
- language structures;
- concepts;
- facts;
- calculation strategies and methods;

used in any particular year are taught, and how these build on previous learning and contribute to future learning.

The Busy Ant Maths scheme of work has been written in the belief that being able to calculate successfully is being able to:

- have a confident and competent understanding of numbers and the number system;
- have instant recall of a set of basic number facts;
- use a range of mental calculation strategies effectively, efficiently and flexibly;
- use a range of written calculation methods accurately and appropriately;
- use and apply all of the above in order to solve problems and reason mathematically.

In order for children to have acquired, understood and mastered the above, they must be taught:

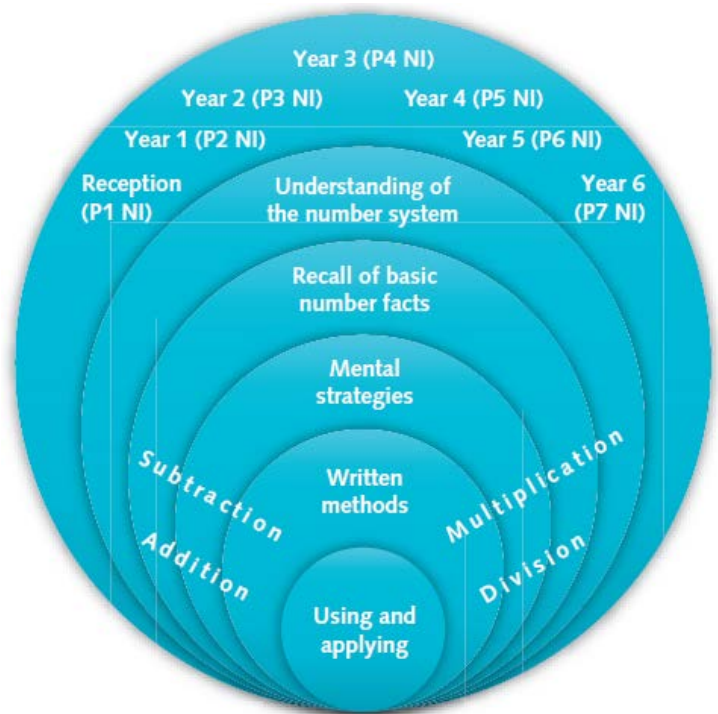
- using a core set of representations, models and images to expose important mathematical structures and ideas;
- precise and accurate language in mathematics to capture, connect and apply mathematical concepts.

The importance of making connections

Although the Busy Ant Maths Calculation Policy is organised by year group, then by operation, and finally by the areas outlined above, it is also vital to stress the importance of the:

- interconnectivity between year groups;
- the four operations;
- pupils' conceptual understanding;
- fluency in number facts;
- mental strategies and written methods;
- the pupils' ability to reason mathematically and solve problems.

This calculation policy not only aims to show the continuity and progression of calculations within the Busy Ant Maths scheme of work across the primary phase for each of the four operations, it also aims to emphasise the important link between the development of children's mental calculation strategies and the teaching of written calculation methods.



Mental recall and strategies, and formal written methods must be seen as complementary to each other. Every written method has a component of mental processing so the two must constantly be developed in conjunction with each other. Pupils' mental facilities with number should be refined as they move through Key stage 2 and not focus exclusively on the written methods of calculation.

The ultimate goal of Busy Ant Maths scheme of work is to ensure that children are confident and competent in their calculation skills, and are able to use and apply these skills in the real world as autonomous problem solvers.

Reception (PI NI)

NUMBER

Counting and understanding numbers

To add, subtract, multiply and divide successfully, pupils need to:

- 1a Count on and back in ones, starting from any number from 0 to 10, then to 20.
- 1b Count objects, actions and sounds from 0 to 10, then to 20.
- 1c Recognise the number of objects presented in familiar patterns up to 10 without counting (subitise).
- 1d Understand that zero represents none of something.
- 1e Estimate a group of objects and check by counting.
- 1f Understand and use ordinal numbers from 1st to 10th in different contexts.
- 1g Compose and decompose numbers to 10.
- 1h Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.
- 1i Count beyond 20, recognising the pattern of the counting system.

Reading and numbers

To add, subtract, multiply and divide successfully, pupils need to:

- 2a Read and write numerals from 0 to 10, then to 20.

Comparing and ordering numbers

To add, subtract, multiply and divide successfully, pupils need to:

- 3a Understand the relative size of quantities to compare numbers from 0 to 10, then to 20.
- 3b Understand the relative size of quantities to order numbers from 0 to 10, then to 20.

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- 4a Understand addition as combining two sets.
- 4b Understand addition as counting on.
- 4e Find 1 more and 1 less than a number from 1 to 10.
- 4f Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

number, count, tens, ones, zero, one, two, ... twenty..., larger, how many, how much, how many more to make, altogether, start at, count forwards, count on, more, 1 more, start, then, now, add, plus, double, and, makes, sum, total, is the same as, is equal to, equal, equals, number sentence

Reason mathematically and solve problems

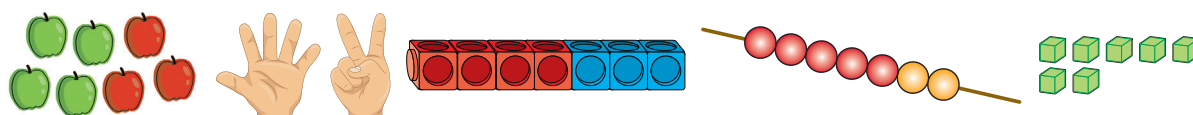
Pupils can be encouraged to think, reason and work mathematically through teaching that promotes the following characteristics:

- Classifying
- Reasoning
- Generalising
- Convincing
- Communicating
- Problem-solving
- Connecting

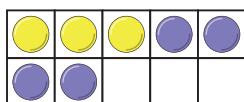
Further guidance on thinking, reasoning and working mathematically, including a list of prompting questions to draw out each of the above characteristics, can be found in the Introduction (pages 19 and 20) of Busy Ant Maths 2nd edition Teacher's Guide Foundation.

Mental strategies

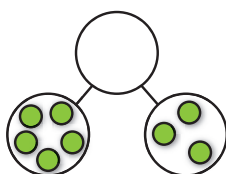
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, Base 10, a selection of different small-world resources (e.g. people, animals and seahorse, transport) and other suitable counting apparatus



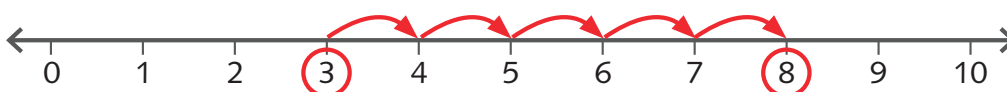
- ten frames



- part-whole models



- number tracks and/or number lines



- Realise the effect of adding zero
- Recall doubles of all numbers to 10
- Begin to understand that addition can be done in any order
- Begin to read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- Begin to recognise and use patterns of similar calculations (e.g. $5 + 0 = 5$, $4 + 1 = 5$, $3 + 2 = 5$)
- Begin to understand and use the inverse relationship between addition and subtraction

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- 4c Understand subtraction as take away.
- 4d Understand subtraction as counting back.
- 4e Find 1 more and 1 less than a number from 1 to 10.
- 4f Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

number, count, tens, ones, zero, one, two, ... twenty..., take away, how many are left, how much is left, how many have gone, how many fewer is... than..., start at, count backwards, count back, count forwards, count on, count up, less, 1 less, start, then, now, subtract, minus, leaves, is the same as, is equal to, equal, equals, number sentence

Reason mathematically and solve problems

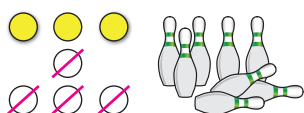
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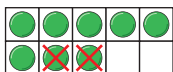
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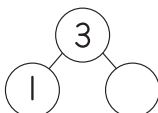
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, Base 10, a selection of different small-world resources (e.g. people, animals and seahorse, transport) and other suitable counting apparatus



- ten frames



- part-whole models



- number tracks and/or number lines



- Realise the effect of subtracting zero
- Begin to understand that subtraction cannot be done in any order
- Begin to read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs
- Begin to recognise and use patterns of similar calculations (e.g. $5 - 0 = 5$, $5 - 1 = 4$, $5 - 3 = 2$...)
- Begin to understand and use the inverse relationship between addition and subtraction

MULTIPLICATION

Although not formally taught in Reception (P1 NI), pupils are introduced to multiplication in Busy Ant Maths 2nd edition Foundation as part of the Number sub-strand: Counting and understanding number.

Fundamental concepts related to multiplication are also included as part of the following *Statutory Framework*: Early Learning Goals in mathematics.

- Numerical Patterns: Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- 1h Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

number, count, tens, ones, zero, one, two, ... twenty..., how many, how much, groups of, same size, equal groups, in each group, double, twice, altogether, is the same as, is equal to, equal, equals, number sentence

Reason mathematically and solve problems

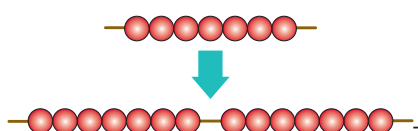
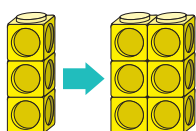
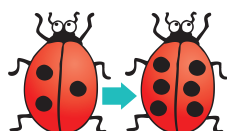
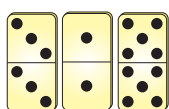
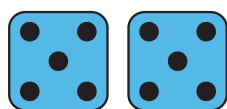
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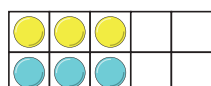
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Mental strategies

- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, a selection of different small-world resources (e.g. people, animals and sealife, transport) and other suitable counting apparatus



- ten frames



- part-whole models



- Understand double means 'twice as many'
- Begin to understand the link between repeated addition and doubling

DIVISION

Although not formally taught in Reception (P1 NI), pupils are introduced to division in Busy Ant Maths 2nd edition Foundation as part of the Number sub-strand: Counting and understanding number.

Fundamental concepts related to division are also included as part of the following *Statutory Framework*: Early Learning Goals in mathematics.

- Numerical Patterns: Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- 1h Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

number, count, tens, ones, zero, one, two, ... twenty..., how many, how much, groups of, fair, unfair, share, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into ... equal parts, divided between, divided equally between, divided evenly between, half, halve, is the same as, is equal to, equal, equals

Reason mathematically and solve problems

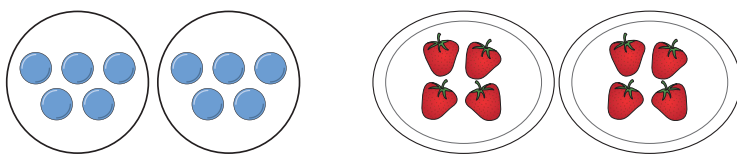
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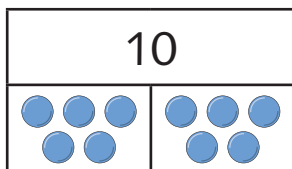
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Mental strategies

- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, a selection of different small-world resources (e.g. people, animals and seafife, transport) and other suitable counting apparatus



- bar models



- Understand the difference between fair (equal) and unfair (unequal) sharing
- Begin to understand division as sharing
- Begin to understand the idea of half as being one of two equal parts

Year 1 (P2 NI)

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- count, read and write numbers from 1 to 20 in numerals and words
- count, read and write numbers to 100 in numerals
- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count in multiples of twos, fives and tens
- given a number, identify one more and one less
- compare and order numbers to at least 20
- identify and represent numbers using objects and pictorial representations, including the number line, and use the language of: equal to, more than, less than (fewer), most, least

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- understand addition as combining two or more groups of objects
- understand addition as counting on
- represent and use number bonds within 20
- add one-digit and two-digit numbers to 20, including zero
- realise the effect of adding zero
- recall doubles of all numbers to 10
- understand that addition can be done in any order
- read, write and interpret mathematical statements involving addition (+) and equals (=) signs

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

tens, ones, addition, add, plus, more, 1 more, 2 more ... 10 more ..., start, then, now, how many, how much, how many more to make, count on, count forwards, double, altogether, sum, total, larger, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

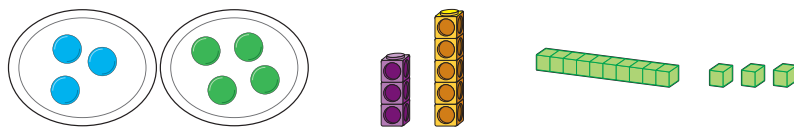
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $16 = \square + 7$
- solve one-step problems that involve addition in familiar contexts, e.g. money

Year 1 (P2 NI)

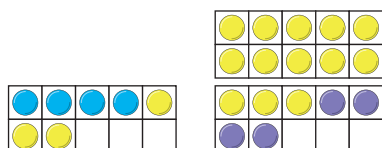
ADDITION Continued

Mental strategies

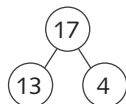
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, Base 10 and other suitable counting apparatus



- ten frames



- part-whole models



- bar models



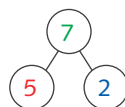
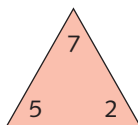
- number tracks and/or number lines



- 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- trios/fact families
 - $5 + 2 = 7$
 - $2 + 5 = 7$
 - $7 - 2 = 5$
 - $7 - 5 = 2$



Year 1 (P2 NI)

ADDITION Continued

Mental strategies continued

– addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Identify near doubles, using doubles already known (e.g. $6 + 5$)
- Recognise and use patterns of similar calculations (e.g. $10 + 0 = 10$, $9 + 1 = 10$, $8 + 2 = 10$...)
- Understand and use the inverse relationship between addition and subtraction

Year 1 (P2 NI)

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- understand subtraction as 'taking away' (counting back)
- understand subtraction as 'finding the difference' (counting up)
- represent and use subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero
- realise the effect of subtracting zero
- understand that subtraction cannot be done in any order
- read, write and interpret mathematical statements involving subtraction ($-$) and equals ($=$) signs

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

tens, ones, take away, how many are left, how much is left, subtraction, subtract, minus, difference, difference between, less, 1 less, 2 less ... 10 less ... , fewer, how many more is... than..., leaves, start, then, now, count on, count back, count backwards, count up, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family

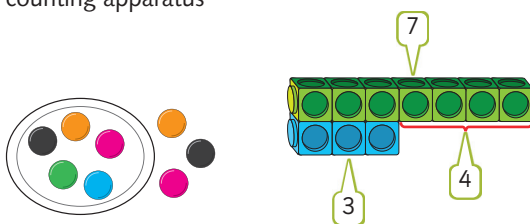
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

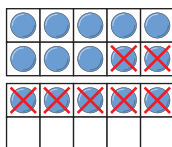
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$
- solve one-step problems that involve subtraction in familiar contexts, e.g. money

Mental strategies

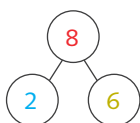
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, Base 10 and other suitable counting apparatus



- ten frames



- part-whole models

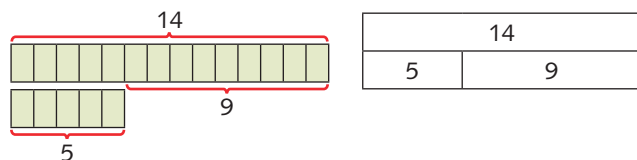


Year 1 (P2 NI)

SUBTRACTION Continued

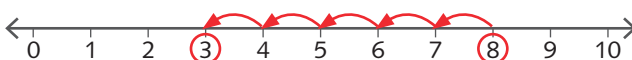
Mental strategies continued

- bar models

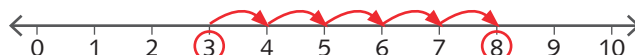


- number tracks and/or number lines

counting back:



counting on / counting up:



- 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
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91	92	93	94	95	96	97	98	99	100

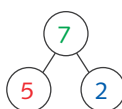
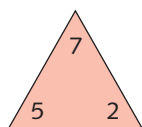
- trios/fact families

$$5 + 2 = 7$$

$$2 + 5 = 7$$

$$7 - 2 = 5$$

$$7 - 5 = 2$$



- addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
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4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Recognise and use patterns of similar calculations (e.g. $10 - 0 = 10$, $10 - 1 = 9$, $10 - 2 = 8$)
- Understand and use the inverse relationship between addition and subtraction

Year 1 (P2 NI)

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- understand multiplication through grouping small quantities
- understand the link between multiplication and doubling

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

number, count, tens, ones, how many, how much, groups of, same size, equal groups, plus, add, repeated addition, in each group, double, twice, array, altogether, is the same as, is equal to, equal, equals, number sentence

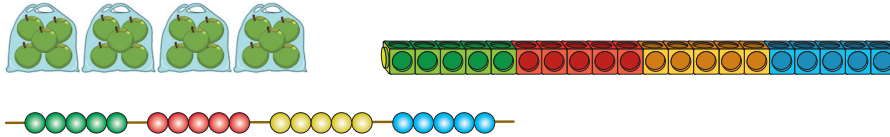
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- solve one-step problems that involve multiplication in familiar contexts

Mental strategies

- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes and other suitable counting apparatus



- ten frames



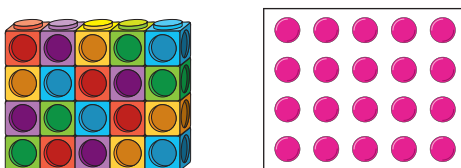
- bar models



- number lines



- arrays



- Make connections between arrays, number patterns and counting in steps of a constant size

Year 1 (P2 NI)

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- understand division through sharing small quantities
- understand the link between division and halving

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

number, count, tens, ones, how many, how much, groups of, fair, unfair, share, equal groups of, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into, divide, divided between, divided equally between, divided evenly between, half, halve, is the same as, is equal to, equal, equals

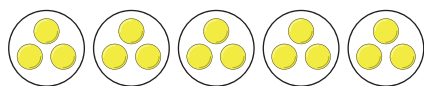
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

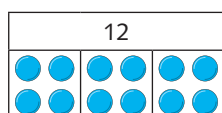
- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- solve one-step problems that involve division in familiar contexts

Mental strategies

- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes and other suitable counting apparatus



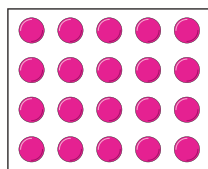
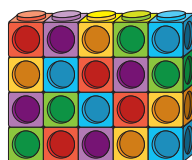
- bar models



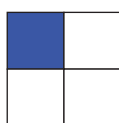
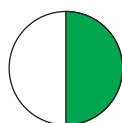
- number lines



- arrays



- shapes



- Make connections between arrays, number patterns and counting in steps of a constant size

Year 2 (P3 NI)

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers to at least 100 in numerals and in words
- count in steps of 2, 3, and 5 from 0, and in tens from any number, forwards and backwards
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use $<$, $>$ and $=$ signs

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- recall and use addition facts to 20 fluently, and derive and use related facts up to 100, including adding two multiples of 10, e.g. $30 + 50$
- add numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- record addition in columns to support place value and prepare for the formal written method with larger numbers

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

hundreds, tens, ones, partition, regroup, combine, addition, add, plus, more, 1 more, 2 more ... 10 more ..., start, then, now, how many, how much, how many more to make, count on, count forwards, double, altogether, sum, total, larger, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, commutative

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

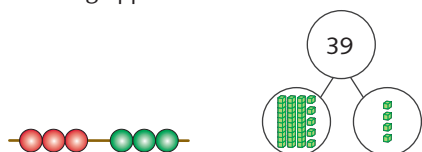
- solve problems with addition:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods

Year 2 (P3 NI)

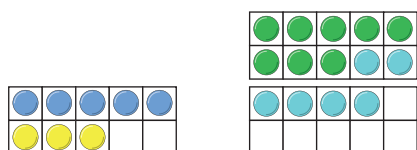
ADDITION Continued

Mental strategies

- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, Base 10 and other suitable counting apparatus



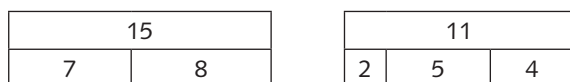
- ten frames



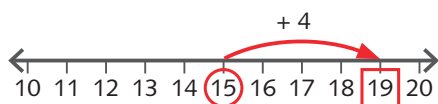
- part-whole models



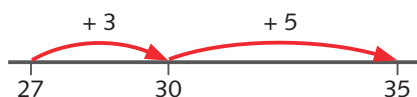
- bar models



- number lines



- empty number lines



- 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Year 2 (P3 NI)

ADDITION Continued

Mental strategies continued

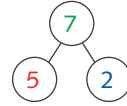
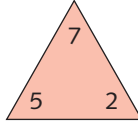
- trios/fact families

$$5 + 2 = 7$$

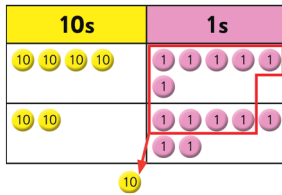
$$2 + 5 = 7$$

$$7 - 2 = 5$$

$$7 - 5 = 2$$



- place value counters and/or place value charts
 $46 + 27 = 73$



- addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Use knowledge that addition can be done in any order (commutative), e.g.
 - put the larger number first and count on in tens or ones
- add three small numbers by putting the largest number first and/or find a pair totalling 10
- Partition additions into tens and ones, then recombine, e.g.

$$38 + 25 = 30 + 20 + 8 + 5$$

$$= 50 + 13$$

$$= 63$$

$$38 + 25 = 38 + 20 + 5$$

$$= 58 + 5$$

$$= 63$$
- Identify near doubles, using doubles already known (e.g. $7 + 8$, $30 + 31$)
- Add a 'near multiple of 10' to a two-digit number by adding 10, 20, 30 and adjusting
- Recognise and use patterns of similar calculations (e.g. $10 + 0 = 10$, $9 + 1 = 10$, $8 + 2 = 10$...)
- Understand and use the inverse relationship between addition and subtraction

Year 2 (P3 NI)

ADDITION Continued

Written methods

- Add two two-digit numbers: TO + TO (beginning with calculations that do not require regrouping and where answers do not exceed 100)

$$38 + 25$$

Link between partitioning and written methods

$$\begin{array}{r} 30 + 8 \\ + 20 + 5 \\ \hline 50 + 13 = 63 \end{array}$$

leads to

Expanded written method

$$\begin{array}{r} 38 \\ + 25 \\ \hline 13 \\ 50 \\ \hline 63 \end{array}$$

leads to

Formal written of columnar addition

$$\begin{array}{r} 38 \\ + 25 \\ \hline 63 \\ 1 \end{array}$$

Record addition calculations in columns to support place value and prepare for the formal written method of columnar addition with larger numbers.

The first stage in the written method shows separately the addition of the ones to the ones and the tens to the tens. The partial sums are then added together to find the total. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method.

The addition of the tens in the calculation $38 + 25$ is described in the words 'thirty add twenty equals fifty', stressing the link to the related fact 'three add two equals five'.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the expanded written methods, before progressing onto the formal written method, will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Year 2 (P3 NI)

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100, including subtracting two multiples of 10, e.g. $80 - 30$
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- record subtraction in columns to support place value and prepare for the formal written method with larger numbers

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

tens, ones, partition, exchange, take away, how many are left, how much is left, subtraction, subtract, minus, difference, difference between, less, 1 less, 2 less ... 10 less ..., fewer, how many more is... than..., leaves, start, then, now, count on, count back, count backwards, count up, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship

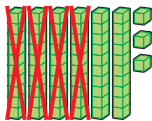
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

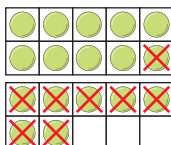
- solve problems with subtraction:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods

Mental strategies

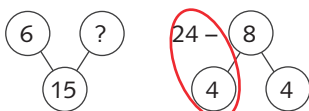
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes, Base 10 and other suitable counting apparatus



- ten frame(s)



- part-whole model

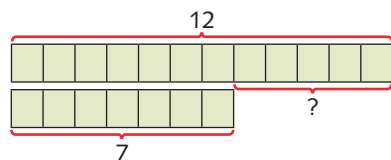


Year 2 (P3 NI)

SUBTRACTION Continued

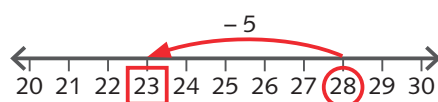
Mental strategies continued

- bar models

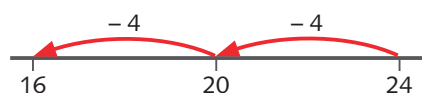


14	
9	?

- number lines



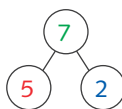
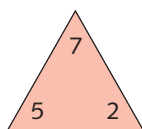
- empty number lines



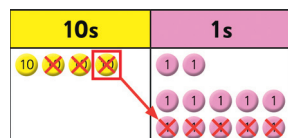
- 1-100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- trios/fact families
 - $5 + 2 = 7$
 - $2 + 5 = 7$
 - $7 - 2 = 5$
 - $7 - 5 = 2$



- place value counters and/or place value charts
 - $42 - 25 = 17$



Year 2 (P3 NI)

Mental strategies continued

– addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Find a small difference by counting up from the smaller to the larger number, e.g. $51 - 4$
- Subtract a 'near multiple of 10' from a two-digit number by subtracting 10, 20, 30 and adjusting
- Recognise and use patterns of similar calculations (e.g. $10 - 0 = 10$, $10 - 1 = 9$, $10 - 2 = 8$...)
- Understand and use the inverse relationship between addition and subtraction
- Use partitioning, e.g.

$$\begin{aligned}
 52 - 28 &= 52 - 20 - 8 \\
 &= 32 - 8 \\
 &= 24
 \end{aligned}$$

Written methods

- Subtract two two-digit numbers: TO – TO (beginning with calculations that do not require decomposition/exchanging)

$$52 - 28$$

Expanded written method

$$\begin{array}{r}
 \overset{40}{\cancel{50}} \overset{12}{\cancel{2}} \\
 - 20 \quad 8 \\
 \hline
 20 \quad 4 \\
 20 + 4 = 24
 \end{array}$$



Formal written of columnar subtraction

$$\begin{array}{r}
 \overset{4}{\cancel{5}} \overset{12}{\cancel{2}} \\
 - 20 \quad 8 \\
 \hline
 2 \quad 4
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 \overset{4}{\cancel{5}} \overset{1}{\cancel{2}} \\
 - 20 \quad 8 \\
 \hline
 2 \quad 4
 \end{array}$$

We can also write the exchanged values like this.

Record subtraction calculations in columns to support place value and prepare for formal written methods of columnar subtraction with larger numbers.

The first stage in the expanded written method is to partition both numbers into tens and ones. Then separately subtract the ones from the ones and the tens from the tens.

In this example the ones to be subtracted are larger than the ones you are subtracting from so 1 ten needs to be exchanged for 10 ones.

In the expanded written method, you exchange one of the 5 tens for 10 ones, crossing out 50 and writing a superscript 40, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 subtract 8 and 40 subtract 20.

For the formal written method, you cross out 5, writing a superscript 4, and cross out 2, writing a superscript 12.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of columnar subtraction (decomposition/exchanging), will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Year 2 (P3 NI)

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- recognise multiplication as repeated addition
- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication ($\times$) and equals (=) signs

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

how many, how much, groups of, same size, equal groups, plus, add, repeated addition, in each group, lots of, groups of, counting in steps of ..., multiplication, times, multiply, multiplied by, product, multiple of, once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition, array, row, column, double, twice, altogether, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

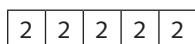
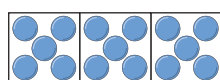
- solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

Mental strategies

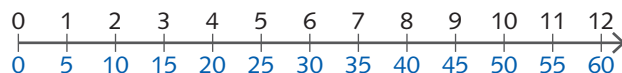
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes and other suitable counting apparatus



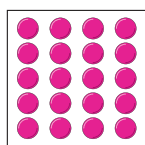
- bar models



- number lines



- arrays



$$4 \times 5 = 20$$

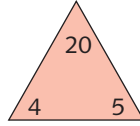
$$5 \times 4 = 20$$

Year 2 (P3 NI)

MULTIPLICATION Continued

Mental strategies continued

- trios $4 \times 5 = 20$
 $5 \times 4 = 20$
 $20 \div 5 = 4$
 $20 \div 4 = 5$



- multiplication and division table

x	2	5	10
1	2	5	10
2	4	10	20
3	6	15	30
4	8	20	40
5	10	25	50
6	12	30	60
7	14	35	70
8	16	40	80
9	18	45	90
10	20	50	100
11	22	55	110
12	24	60	120

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division, including doubling and halving

Year 2 (P3 NI)

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- recognise division as grouping or sharing
- recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- understand the link between division and fractions, and find fractions of a length, shape, set of objects or quantity
- calculate mathematical statements for division within the multiplication tables and write them using the division ($\div$) and equals (=) signs

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

how many, how much, groups of, fair, unfair, share, equal groups of, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into, division, divide, divided between, divided equally between, divided evenly between, divided by, divided into, left, left over, half, halve, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship

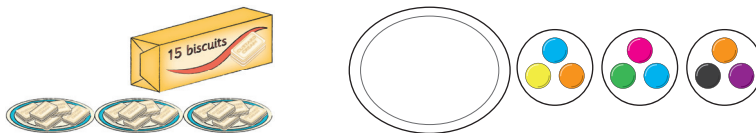
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

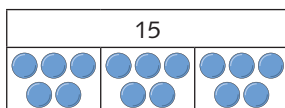
- solve problems involving division, using materials, arrays, repeated addition and subtraction, mental methods, and division facts, including problems in contexts

Mental strategies

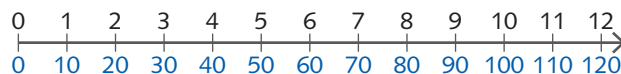
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes and other suitable counting apparatus



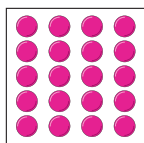
- bar models



- number lines



- arrays



$$20 \div 5 = 4$$

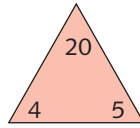
$$20 \div 4 = 5$$

Year 2 (P3 NI)

DIVISION Continued

Mental strategies continued

- trios $4 \times 5 = 20$
 $5 \times 4 = 20$
 $20 \div 5 = 4$
 $20 \div 4 = 5$



- multiplication and division table

×	2	5	10
1	2	5	10
2	4	10	20
3	6	15	30
4	8	20	40
5	10	25	50
6	12	30	60
7	14	35	70
8	16	40	80
9	18	45	90
10	20	50	100
11	22	55	110
12	24	60	120

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division, including doubling and halving

Year 3 (P4 NI)

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers up to 1000 in numerals and in words
- count from 0 in multiples of 1, 2, 3, 4, 5, 8, 10, 50 and 100, forwards and backwards
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- identify, represent and estimate numbers using different representations
- find 10 or 100 more or less than a given number
- compare and order numbers up to 1000

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- continue to recall and use addition facts to 20 fluently, and derive and use related facts up to 100, e.g. $130 + 50 = 180$
- continue to add numbers mentally, including:
 - two two-digit numbers
 - three or more one-digit numbers
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add numbers with up to three digits, using the formal written method of columnar addition
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

hundreds, tens, ones, partition, regroup, combine, addition, add, plus, more, 1 more, 2 more ... 10 more, 100 more, how many, how much, how many more to make, count on, count forwards, double, altogether, sum, total, larger, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, commutative

Reason mathematically and solve problems

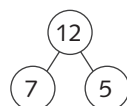
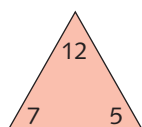
Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve problems, including missing number problems, using number facts, place value, and more complex addition

Mental strategies

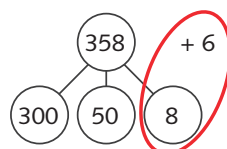
- Use of core representations, models and images:

- trios/fact families
 - $7 + 5 = 12$
 - $5 + 7 = 12$
 - $12 - 5 = 7$
 - $12 - 7 = 5$

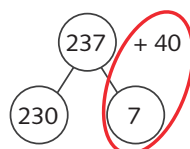


- part-whole models

$$358 + 6 = 164$$



$$237 + 40 = 277$$



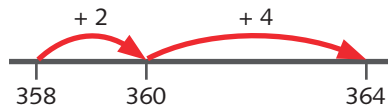
Year 3 (P4 NI)

ADDITION Continued

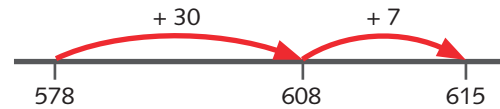
Mental strategies continued

- empty number lines

$$358 + 6 = 364$$

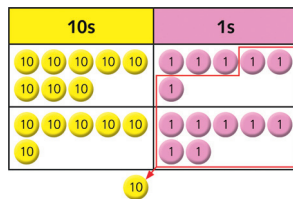


$$578 + 37 = 615$$



- place value counters and/or place value charts

$$86 + 67 = 153$$



$$387 + 456 = 843$$



- multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Recognise and use the inverse relationship between addition and subtraction
- Use knowledge that addition can be done in any order (commutative), e.g.
 - put the larger number first and count on in steps of 1, 10 or 100
 - partition additions into hundreds, tens and ones, then recombine, e.g. $75 + 56 = 75 + 50 + 6$

$$= 125 + 6$$

$$= 131$$
- Identify near doubles, using doubles already known, e.g. $70 + 71$
- Add the nearest multiple of 10 or 100, and adjust
- Use patterns of similar calculations, e.g. $13 + 5 = 18$ and $130 + 50 = 180$
- Use knowledge of the associative law when adding more than two numbers, e.g. $4 + 7 + 6 = (4 + 6) + 7$

$$= 10 + 7$$

$$= 17$$

Year 3 (P4 NI)

ADDITION Continued

Written methods

- Add numbers with up to three digits (HTO + HTO)
- Estimate and check the answer to a calculation

Expanded written method

$$548 + 387$$

$$\begin{array}{r} 548 \\ + 387 \\ \hline 15 \\ 120 \\ 800 \\ \hline 935 \end{array}$$

The first stage in the expanded written method shows separately the addition of the ones to the ones, the tens to the tens and the hundreds to the hundreds. The partial sums are then added together to find the total. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method of columnar addition.

The addition of the tens in the calculation $548 + 387$ is described in the words 'forty add eighty equals one hundred and twenty', stressing the link to the related fact 'four add eight equals twelve'. The addition of the hundreds is described as 'five hundred add three hundred equals eight hundred', stressing the link to the related fact 'five add three equals eight'.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of columnar addition, will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Formal written method of columnar addition

$$548 + 387$$

$$\begin{array}{r} 548 \\ + 387 \\ \hline 935 \\ 11 \end{array}$$

The expanded written method leads to the formal written method of columnar addition so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Regrouped digits are recorded below the line, using the phrases 'regroup 10 ones into 1 ten', and/or 'regroup 10 tens into 1 hundred'.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 3 (P4 NI)

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- continue to recall and use subtraction facts to 20 fluently, and derive and use related facts, e.g. $120 - 70 = 50$
- continue to subtract numbers mentally, including:
 - two two-digit numbers
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- subtract numbers with up to three digits, using the formal written method of columnar subtraction
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

hundreds, tens, ones, partition, exchange, take away, how many are left, how much is left, subtraction, subtract, minus, difference, difference between, less, 1 less, 2 less ... 10 less, 100 less, fewer, how many more is... than..., leaves, count on, count back, count backwards, count up, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship

Reason mathematically and solve problems

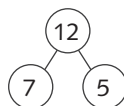
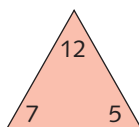
Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve problems, including missing number problems, using number facts, place value, and more complex subtraction

Mental strategies

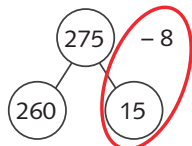
- Use of core representations, models and images:

- trios/fact families
 - $7 + 5 = 12$
 - $5 + 7 = 12$
 - $12 - 5 = 7$
 - $12 - 7 = 5$

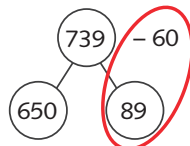


- part-whole models

$$275 - 8 = 267$$

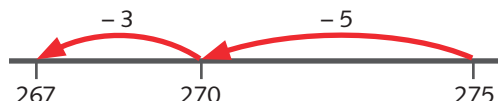


$$739 - 60 = 679$$

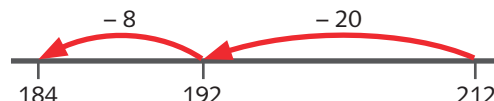


- empty number lines

$$275 - 8 = 267$$

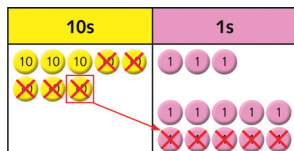


$$212 - 28 = 184$$

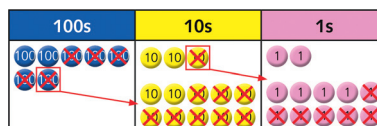


- place value counters and/or place value charts

$$83 - 45 = 38$$



$$732 - 486 = 246$$



Year 3 (P4 NI)

SUBTRACTION Continued

Mental strategies continued

– multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Recognise and use the inverse relationship between addition and subtraction
- Find a difference by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10 or 100, and adjust
- Use patterns of similar calculations, e.g. $18 - 5 = 13$ and $180 - 50 = 130$
- Use partitioning, e.g. $73 - 46 = 73 - 40 - 6$
 $= 33 - 6$
 $= 27$

Written methods

- Subtract numbers with up to three digits (HTO – HTO)
- Estimate and check the answer to a calculation

Expanded written method

$$582 - 237$$

$$\begin{array}{r}
 500 \quad \overset{70}{\cancel{80}} \quad \overset{12}{\cancel{2}} \\
 - 200 \quad 30 \quad 7 \\
 \hline
 300 \quad 40 \quad 5
 \end{array}$$

$$300 + 40 + 5 = 345$$

The first stage in the expanded written method is to partition both numbers into hundreds, tens and ones. Then separately subtract the ones from the ones, the tens from the tens and the hundreds from the hundreds.

In this example the ones to be subtracted are larger than the ones you are subtracting from so 1 ten needs to be exchanged for 10 ones. You exchange one of the 8 tens for 10 ones, crossing out 80 and writing a superscript 70, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 subtract 7, 70 subtract 30 and finally 500 subtract 200.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of columnar subtraction (decomposition/exchanging), will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Year 3 (P4 NI)

Written methods continued

Formal written method of columnar subtraction (decomposition/exchanging)

$$582 - 237$$

$$\begin{array}{r} 5 \overset{7}{\cancel{8}} \overset{12}{\cancel{2}} \\ - 237 \\ \hline 345 \end{array} \quad \text{or} \quad \begin{array}{r} 5 \overset{7}{\cancel{8}} \overset{1}{\cancel{2}} \\ - 237 \\ \hline 345 \end{array}$$

We can also write the exchanged values like this.

The expanded written method leads to the formal written method of columnar subtraction (decomposition/exchanging) so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Start by subtracting the least significant digits first, i.e. the ones, then the tens, and finally the hundreds. Refer to subtracting the tens, for example, by saying 'seventy subtract thirty', not 'seven subtract three'.

In this example the ones to be subtracted are larger than the ones you are subtracting from so 1 ten needs to be exchanged for 10 ones.

You exchange one of the 8 tens for 10 ones, crossing out 8 and writing a superscript 7, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 subtract 7, 70 subtract 30 and finally 500 subtract 200.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 3 (P4 NI)

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of multiplication facts for the 2, 5 and 10 multiplication tables
- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- use known multiplication facts to derive related facts involving multiples of 10, e.g. $2 \times 30 = 60$
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods and progressing to formal written methods
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

hundreds, tens, ones, partition, decompose, regroup, combine, how many, how much, groups of, same size, equal groups, plus, add, repeated addition, in each group, lots of, groups of, counting in steps of ..., multiplication, times, multiply, multiplied by, product, factor, multiple of, once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition, array, row, column, double, twice, altogether, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, commutative

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

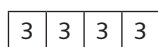
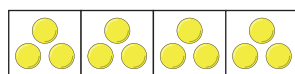
- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Mental strategies

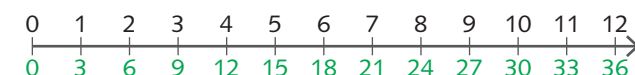
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes and other suitable counting apparatus



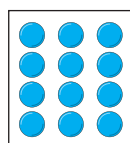
- bar models



- number lines



- arrays



$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

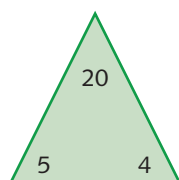
Year 3 (P4 NI)

Mental strategies continued

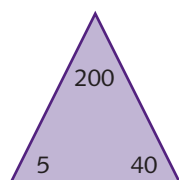
– 1 – 100 number squares

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

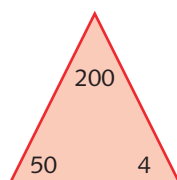
– trios



$$\begin{aligned} 5 \times 4 &= 20 \\ 4 \times 5 &= 20 \\ 20 \div 4 &= 5 \\ 20 \div 5 &= 4 \end{aligned}$$



$$\begin{aligned} 5 \times 40 &= 200 \\ 40 \times 5 &= 200 \\ 200 \div 40 &= 5 \\ 200 \div 5 &= 40 \end{aligned}$$



$$\begin{aligned} 50 \times 4 &= 200 \\ 4 \times 50 &= 200 \\ 200 \div 4 &= 50 \\ 200 \div 50 &= 4 \end{aligned}$$

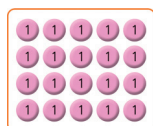
– multiplication grids

×	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
8	8	16	24	32	40	48	56	64	72	80	88	96
10	10	20	30	40	50	60	70	80	90	100	110	120

×	1	2	3	4	5	6	7	8	9	10	11	12
20	20	40	60	80	100	120	140	160	180	200	220	240
30	30	60	90	120	150	180	210	240	270	300	330	360
40	40	80	120	160	200	240	280	320	360	400	440	480
50	50	100	150	200	250	300	350	400	450	500	550	600
80	80	160	240	320	400	480	560	640	720	800	880	960
100	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200

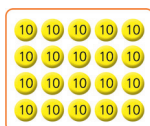
– place value counters

$$5 \times 4 = 20$$



10 times larger

$$5 \times 40 = 200$$



$$36 \times 4 = 144$$

×	30	6
4	10 10 10 10 10 10 10 10 10 10 10 10	1 1

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division
- Use doubling, e.g. connect the 2, 4 and 8 multiplication tables

Year 3 (P4 NI)

MULTIPLICATION Continued

Mental strategies continued

- Use the 'key multiplication facts' of $\times 1$, $\times 2$, $\times 5$, and $\times 10$ to work out the answers to unknown multiplication facts, e.g. $7 \times 4 = (5 \times 4) + (2 \times 4)$
 $= 20 + 8$
 $= 28$
- Use patterns of similar calculations, e.g. $8 \times 6 = 48$ and $8 \times 60 = 480$
- Show that multiplication of two or more numbers can be done in any order (commutative), e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12$
 $= 20 \times 12$
 $= 240$
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit number by a one-digit number, $63 \times 8 = (60 \times 8) + (3 \times 8)$
 $= 480 + 24$
 $= 504$

Written methods

- Short multiplication:
 - Multiply a two-digit number by a one-digit number (TO $\times$ O)
- Estimate and check the answer to a calculation

Grid method

$$63 \times 8$$

$\times$	60	3
8	480	24

 $= 504$

Expanded written method

$$63 \times 8$$

63	$\times$	8	
		24	(3 $\times$ 8)
480			(60 $\times$ 8)
504			

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the partial calculations: (3 $\times$ 8) and (60 $\times$ 8) may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 63×8 it is 'sixty multiplied by eight', not 'six multiplied by eight', although the relationship 6×8 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of short multiplication, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10, with adding whole numbers mentally, and in their understanding of place value.

Formal written method of short multiplication

$\begin{array}{r} 63 \\ \times 8 \\ \hline 504 \end{array}$	or	$\begin{array}{r} 63 \\ \times 8 \\ \hline 504 \\ \hline \end{array}$
---	----	---

You can also write the regrouped value like this.

The grid method and the expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 3 (P4 NI)

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of division facts for the 2, 5 and 10 multiplication tables
- recall and use division facts for the 3, 4 and 8 multiplication tables
- use known division facts to derive related facts involving multiples of 10, e.g. $60 \div 3 = 20$
- write and calculate mathematical statements for division using the multiplication tables that they know
- develop reliable written methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written method of short division (without a remainder)
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

tens, ones, partition, decompose, regroup, exchange, how many, how much, groups of, share, equal groups of, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into, division, divide, divided between, divided equally between, divided evenly between, divided by, divided into, dividend, divisor, quotient, left, left over, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

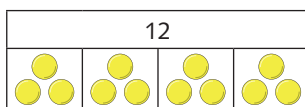
- solve problems, including missing number problems, involving division

Mental strategies

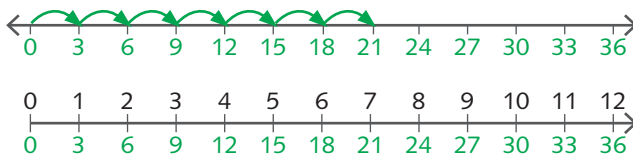
- Use of core representations, models and images:
 - concrete objects/pictorial representations, such as counters, interlocking cubes and other suitable counting apparatus



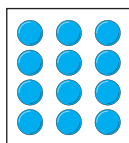
- bar models



- number lines



- arrays



$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

Year 3 (P4 NI)

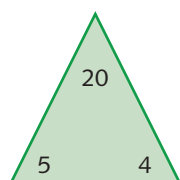
DIVISION Continued

Mental strategies continued

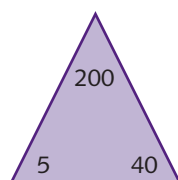
- 1 – 100 number squares

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

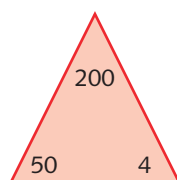
- trios



$$\begin{aligned} 5 \times 4 &= 20 \\ 4 \times 5 &= 20 \\ 20 \div 4 &= 5 \\ 20 \div 5 &= 4 \end{aligned}$$



$$\begin{aligned} 5 \times 40 &= 200 \\ 40 \times 5 &= 200 \\ 200 \div 40 &= 5 \\ 200 \div 5 &= 40 \end{aligned}$$



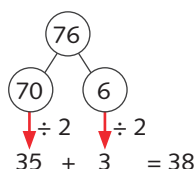
$$\begin{aligned} 50 \times 4 &= 200 \\ 4 \times 50 &= 200 \\ 200 \div 4 &= 50 \\ 200 \div 50 &= 4 \end{aligned}$$

- multiplication grids

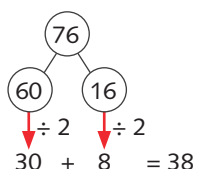
x	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
8	8	16	24	32	40	48	56	64	72	80	88	96
10	10	20	30	40	50	60	70	80	90	100	110	120

x	1	2	3	4	5	6	7	8	9	10	11	12
20	20	40	60	80	100	120	140	160	180	200	220	240
30	30	60	90	120	150	180	210	240	270	300	330	360
40	40	80	120	160	200	240	280	320	360	400	440	480
50	50	100	150	200	250	300	350	400	450	500	550	600
80	80	160	240	320	400	480	560	640	720	800	880	960
100	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200

- part-whole models



or



Year 3 (P4 NI)

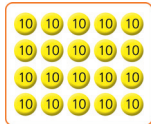
DIVISION Continued

Mental strategies continued

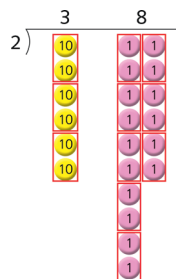
- place value counters

$$200 \div 40 = 5$$

$$200 \div 4 = 50$$



$$76 \div 2 = 38$$



- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division
- Use halving, e.g. find quarters by halving halves
- Use patterns of similar calculations, e.g. $48 \div 8 = 6$ and $480 \div 80 = 6$
- Understand and use the distributive law, e.g. partitioning when dividing a two-digit number by a one-digit number, $92 \div 4 = (80 \div 4) + (12 \div 4)$
 $= 20 + 3$
 $= 23$

Written methods

- Short division (without a remainder):
 - Divide a two-digit number by a one-digit number ($TO \div O$)
- Estimate and check the answer to a calculation

Expanded written method

$$92 \div 4$$

$$\begin{array}{r} 23 \\ 4 \overline{) 92} \\ \underline{- 80} \quad (20 \times 4) \\ 12 \\ \underline{- 12} \quad (3 \times 4) \\ 0 \end{array}$$

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (20×4) and (3×4) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many fours divide into 90 so that the answer is a multiple of 10? (20) There are 20 fours or 80, with 12 remaining. How many fours in 12? (3) So 92 divided by four is 23.'

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of short division, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10, with subtracting whole numbers mentally, and in their understanding of place value.

Formal written method of short division

$$\begin{array}{r} 23 \\ 4 \overline{) 92} \end{array}$$

The expanded written method leads children to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 1 represents the 1 ten that is remaining after 4 has been divided into 90. It is written in front of the 2 to show that 12 now has to be divided by 4.

Year 4 (P5 NI)

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers up to 10 000 in numerals and in words
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- identify, represent and estimate numbers using different representations
- find 10, 100 or 1000 more or less than a given number
- compare and order numbers beyond 1000
- round any number to the nearest 10, 100 or 1000

DECIMALS

To add and subtract successfully, pupils need to:

- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise the place value of each digit in a decimal to two decimal places
- compare and order numbers with the same number of decimal places up to two decimal places
- round decimals with one decimal place to the nearest whole number
- understand the effect of multiplying and dividing a one-digit or two-digit number by 10 and 100

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- consolidate recall of addition facts to 20 and related facts involving multiples of 100 and 1000, e.g. $1300 + 500 = 1800$ and $500 + 1300 = 1800$
- continue to add numbers mentally, including:
 - two two-digit numbers
 - three or more one-digit numbers
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add numbers with up to four digits using the formal written method of columnar addition where appropriate, including calculations involving money, e.g. $£13.56 + £38.54$
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

thousands, hundreds, tens, ones, partition, regroup, combine, addition, add, plus, more, 1 more, 2 more ... 10 more, 100 more, 1000 more, how many, how much, how many more to make, count on, count forwards, double, altogether, sum, total, larger, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, commutative, associative

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve addition two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving decimals to two decimal places

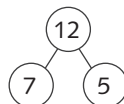
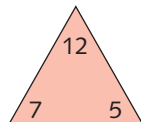
Year 4 (P5 NI)

ADDITION Continued

Mental strategies

- Continue to use core representations, models and images when necessary:

- trios/fact families $7 + 5 = 12$
 $5 + 7 = 12$
 $12 - 5 = 7$
 $12 - 7 = 5$



- multiples of 10 and 100 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

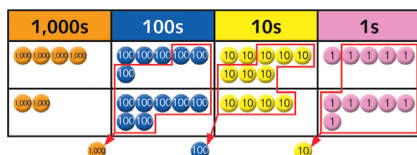
+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- multiples of 100 addition and subtraction number facts

+	0	100	200	300	400	500	600	700	800	900	1,000
0	0	100	200	300	400	500	600	700	800	900	1,000
100	100	200	300	400	500	600	700	800	900	1,000	1,100
200	200	300	400	500	600	700	800	900	1,000	1,100	1,200
300	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300
400	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400
500	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500
600	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600
700	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700
800	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800
900	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900
1,000	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900	2,000

- place value counters and/or place value charts

$$4685 + 2746 = 7431$$



Year 4 (P5 NI)

ADDITION Continued

Mental strategies continued

- Continue to use the relationship between addition and subtraction
- Use knowledge of the commutative law, e.g.
 - put the larger number first and count on in steps of 1, 10 or 100
 - partition additions into hundreds, tens and ones, then recombine, e.g. $356 + 57 = 356 + 50 + 7$
 $= 406 + 7$
 $= 413$
- Identify near doubles, using doubles already known, e.g. $170 + 180$
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. $130 + 50 = 180$ and $1300 + 500 = 1800$
- Use knowledge of the associative law when adding more than two numbers, e.g. $24 + 27 + 16 = (24 + 16) + 27$
 $= 40 + 27$
 $= 67$

Written methods

- Add numbers with up to four digits, including money and measures (ThHTO + ThHTO)
- Estimate and check the answer to a calculation

Formal written method of columnar addition

$$2456 + 5378$$

$$\begin{array}{r} 2456 \\ + 5378 \\ \hline 7834 \\ \hline \end{array}$$

Regrouped digits are recorded below the line, using the phrases 'regroup 10 ones into 1 ten', and/or 'regroup 10 tens into 1 hundred' and/or 'regroup 10 hundreds into 1 thousand'.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar addition.

Year 4 (P5 NI)

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- consolidate recall of subtraction facts to 20 and related facts involving multiples of 100 and 1000, e.g. $1800 - 500 = 1300$ and $1800 - 1300 = 500$
- continue to subtract numbers mentally, including:
 - two two-digit numbers
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- subtract numbers with up to four digits using the formal written method of columnar subtraction where appropriate, including calculations involving money, e.g. $£24.26 - £17.58$
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

hundreds, tens, ones, partition, exchange, take away, how many are left, how much is left, subtraction, subtract, minus, difference, difference between, less, 1 less, 2 less ... 10 less, 100 less, 1000 less, fewer, how many more is... than..., leaves, count on, count back, count backwards, count up, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship

Reason mathematically and solve problems

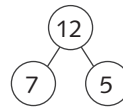
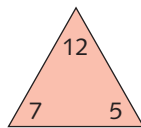
Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving decimals to two decimal places

Mental strategies

- Continue to use core representations, models and images when necessary:

- trios/fact families
 - $7 + 5 = 12$
 - $5 + 7 = 12$
 - $12 - 5 = 7$
 - $12 - 7 = 5$



- multiples of 10 and 100 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

Year 4 (P5 NI)

SUBTRACTION Continued

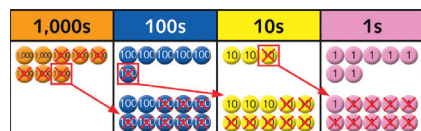
Mental strategies continued

- multiples of 100 addition and subtraction number facts

+	0	100	200	300	400	500	600	700	800	900	1,000
0	0	100	200	300	400	500	600	700	800	900	1,000
100	100	200	300	400	500	600	700	800	900	1,000	1,100
200	200	300	400	500	600	700	800	900	1,000	1,100	1,200
300	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300
400	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400
500	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500
600	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600
700	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700
800	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800
900	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900
1,000	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900	2,000

- place value counters and/or place value charts

$$8637 - 5879 = 2758$$



- Continue to use the relationship between addition and subtraction
- Calculate mentally a difference such as $5005 - 2998$ by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. $18 - 5 = 13$ and $1800 - 500 = 1300$
- Use partitioning, e.g. $456 - 84 = 456 - 80 - 4$

$$= 376 - 4$$

$$= 372$$

Year 4 (P5 NI)

SUBTRACTION Continued

Written methods

- Subtract numbers with up to four digits, including money and measures (ThHTO – ThHTO)
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition/exchanging)

6418 – 2546

$$\begin{array}{r}
 \overset{5}{\cancel{6}} \overset{13}{\cancel{4}} \overset{11}{\cancel{8}} 8 \\
 - 2546 \\
 \hline
 3872
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 \overset{5}{\cancel{6}} \overset{13}{\cancel{4}} \overset{1}{1} 8 \\
 - 2546 \\
 \hline
 3872
 \end{array}$$

You can also write the exchanged values like this.

Start by subtracting the least significant digits first, i.e. the ones, then the tens, then the hundreds and finally the thousands. Refer to subtracting the tens, for example, by saying '11 tens subtract four tens', not '11 subtract four'.

In this example the tens and the hundreds to be subtracted are larger than both the tens and hundreds you are subtracting from so 1 hundred needs to be exchanged for 10 tens, and 1 thousand for 10 hundreds.

The calculation begins 8 subtract 6.

Then you exchange one of the 4 hundreds for 10 tens, crossing out 4 and writing a superscript 3, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 tens subtract 4 tens.

You then exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and writing a superscript 1 in front of the 3 to make 13 hundreds. The calculation then becomes 13 hundreds subtract 5 hundreds.

Then finally 5000 subtract 2000.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar subtraction (decomposition/exchanging).

Year 4 (P5 NI)

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- recall and use multiplication facts for the 6, 7, 9, 11 and 12 multiplication tables
- use known multiplication facts to derive related facts involving multiples of 10 and 100, e.g. $200 \times 3 = 600$
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

thousands, hundreds, tens, ones, tenths, hundredths, partition, decompose, regroup, combine, how many, how much, groups of, same size, equal groups, plus, add, repeated addition, in each group, lots of, groups of, counting in steps of ..., multiplication, times, multiply, multiplied by, product, factor, multiple of, 10 times greater, 100 times greater, repeated addition, array, row, column, double, twice, altogether, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, commutative, associative, decimal, decimal point, place holder

Reason mathematically and solve problems

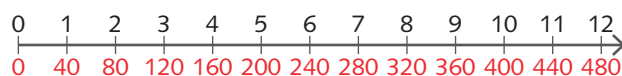
Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems involving multiplying and adding, including using the distributive law and multiply two-digit numbers by one digit, including scaling problems and harder correspondence problems such as n objects are connected to m objects

Mental strategies

- Continue to use core representations, models and images:

- number lines



- arrays



$$6 \times 7 = 42$$

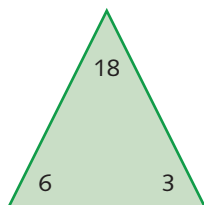
$$7 \times 6 = 42$$

Year 4 (P5 NI)

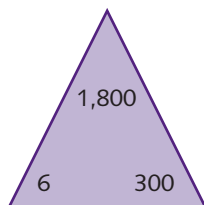
MULTIPLICATION Continued

Mental strategies continued

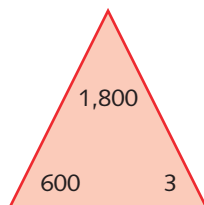
– trios



$$\begin{aligned} 6 \times 3 &= 18 \\ 3 \times 6 &= 18 \\ 18 \div 3 &= 6 \\ 18 \div 6 &= 3 \end{aligned}$$



$$\begin{aligned} 6 \times 300 &= 1,800 \\ 300 \times 6 &= 1,800 \\ 1,800 \div 300 &= 6 \\ 1,800 \div 6 &= 300 \end{aligned}$$



$$\begin{aligned} 600 \times 3 &= 1,800 \\ 3 \times 600 &= 1,800 \\ 1,800 \div 3 &= 600 \\ 1,800 \div 600 &= 3 \end{aligned}$$

– multiplication square to 12 x 12 / multiples of 10 multiplication square

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

x	10	20	30	40	50	60	70	80	90	100	110	120
1	10	20	30	40	50	60	70	80	90	100	110	120
2	20	40	60	80	100	120	140	160	180	200	220	240
3	30	60	90	120	150	180	210	240	270	300	330	360
4	40	80	120	160	200	240	280	320	360	400	440	480
5	50	100	150	200	250	300	350	400	450	500	550	600
6	60	120	180	240	300	360	420	480	540	600	660	720
7	70	140	210	280	350	420	490	560	630	700	770	840
8	80	160	240	320	400	480	560	640	720	800	880	960
9	90	180	270	360	450	540	630	720	810	900	990	1,080
10	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200
11	110	220	330	440	550	660	770	880	990	1,100	1,210	1,320
12	120	240	360	480	600	720	840	960	1,080	1,200	1,320	1,440

– Gattegno charts

$$47 \times 100 = 4700$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

$$368 \times 10 = 3680$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

– place value charts

$$279 \times 10 = 2790$$

1,000s	100s	10s	1s
	2	7	9
2	7	9	0

$$82 \times 100 = 8200$$

1,000s	100s	10s	1s
		8	2
8	2	0	0

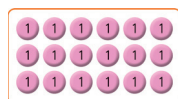
Year 4 (P5 NI)

MULTIPLICATION Continued

Mental strategies continued

- place value counters

$$6 \times 3 = 18$$



100 times larger

$$6 \times 300 = 1800$$



$$543 \times 7$$

x	500	40	3
7	3500	280	21

- Make connections between arrays, number patterns and counting in steps of a constant size
- Continue to use the inverse relationship between multiplication and division
- Continue to use doubling, e.g. connect the 3, 6 and 12 multiplication tables
- Use the 'key multiplication facts' of $\times 1$, $\times 2$, $\times 5$, and $\times 10$ to work out the answers to unknown multiplication facts, e.g. $7 \times 9 = (5 \times 9) + (2 \times 9)$
 $= 45 + 18$
 $= 63$
- Use closely related facts:
 - multiply by 9 or 11 by multiplying by 10 and adjusting
 - develop the $\times 12$ table by adding facts from the $\times 10$ and $\times 2$ table
- Use factors, e.g. $8 \times 14 = 8 \times 2 \times 7$
- Use patterns of similar calculations, e.g. $8 \times 6 = 48$ and $8 \times 60 = 480$
- Understand and use the commutative law
- Understand and use the associative law, e.g. $6 \times 15 = 6 \times (5 \times 3)$
 $= (6 \times 5) \times 3$
 $= 30 \times 3$
 $= 90$
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, e.g.
 $356 \times 7 = (300 \times 7) + (50 \times 7) + (6 \times 7)$
 $= 2100 + 350 + 42$
 $= 2492$

Written methods

- Short multiplication:
 - Multiply a two-digit or three-digit number by a one-digit number (TO $\times$ O/HTO $\times$ O)
- Estimate and check the answer to a calculation

Grid method

$$356 \times 7$$

$\times$	300	50	6	
7	2100	350	42	= 2492

Expanded written method

$$356 \times 7$$

	3	5	6	
$\times$				7
	4	2		(6 $\times$ 7)
	3	5	0	(50 $\times$ 7)
	2	1	0	0 (300 $\times$ 7)
	2	4	9	2

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the partial calculations: (6 $\times$ 7), (50 $\times$ 7) and (300 $\times$ 7) may be recorded to the right of the algorithm.

Year 4 (P5 NI)

MULTIPLICATION Continued

Written methods continued

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 356×7 it is 'fifty multiplied by seven', not 'five multiplied by seven', although the relationship 5×7 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of short multiplication, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with adding whole numbers mentally, and in their understanding of place value.

Formal written method of short multiplication

$$356 \times 7$$

$\begin{array}{r} 3 \ 5 \ 6 \\ \times 7 \\ \hline 2 \ 1 \ 9 \ 2 \end{array}$	or	$\begin{array}{r} 3 \ 5 \ 6 \\ \times 7 \\ \hline 2 \ 1 \ 9 \ 2 \\ \hline \end{array}$
--	----	--

You can also write the regrouped value like this.

The grid method and the expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Year 4 (P5 NI)

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- recall and use division facts for the 6, 7, 9, 11 and 12 multiplication tables
- use known division facts to derive related facts involving multiples of 10 and 100, e.g. $600 \div 3 = 200$
- use place value, known and derived facts to divide mentally, including dividing by 1 and itself
- recognise and use factor pairs in mental calculations
- divide two-digit and three-digit numbers by a one-digit number using formal written layout (without a remainder)
- estimate and check the answer to a calculation, including using the inverse operation

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

hundreds, tens, ones, tenths, hundredths, partition, decompose, regroup, exchange, how many, how much, groups of, share, equal groups of, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into, division, divide, divided between, divided equally between, divided evenly between, divided by, divided into, dividend, divisor, quotient, 10 times smaller, 100 times smaller, left, left over, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, decimal, decimal point, place holder

Reason mathematically and solve problems

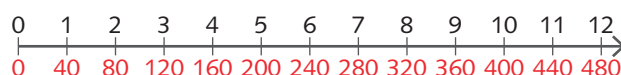
Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving division in contexts, deciding which operations and methods to use and why

Mental strategies

- Continue to use core representations, models and images:

- number lines



- arrays



$$42 \div 7 = 6$$

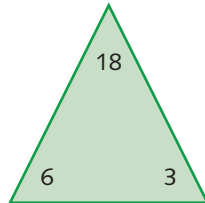
$$42 \div 6 = 7$$

Year 4 (P5 NI)

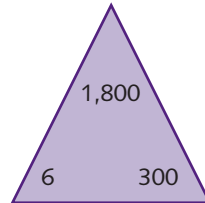
DIVISION Continued

Mental strategies continued

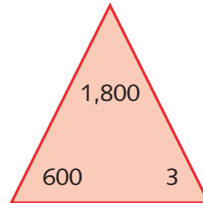
– trios



$$\begin{aligned} 6 \times 3 &= 18 \\ 3 \times 6 &= 18 \\ 18 \div 3 &= 6 \\ 18 \div 6 &= 3 \end{aligned}$$



$$\begin{aligned} 6 \times 300 &= 1,800 \\ 300 \times 6 &= 1,800 \\ 1,800 \div 300 &= 6 \\ 1,800 \div 6 &= 300 \end{aligned}$$



$$\begin{aligned} 600 \times 3 &= 1,800 \\ 3 \times 600 &= 1,800 \\ 1,800 \div 3 &= 600 \\ 1,800 \div 600 &= 3 \end{aligned}$$

– multiplication square to 12 × 12 / multiples of 10 multiplication square

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

×	10	20	30	40	50	60	70	80	90	100	110	120
1	10	20	30	40	50	60	70	80	90	100	110	120
2	20	40	60	80	100	120	140	160	180	200	220	240
3	30	60	90	120	150	180	210	240	270	300	330	360
4	40	80	120	160	200	240	280	320	360	400	440	480
5	50	100	150	200	250	300	350	400	450	500	550	600
6	60	120	180	240	300	360	420	480	540	600	660	720
7	70	140	210	280	350	420	490	560	630	700	770	840
8	80	160	240	320	400	480	560	640	720	800	880	960
9	90	180	270	360	450	540	630	720	810	900	990	1,080
10	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200
11	110	220	330	440	550	660	770	880	990	1,100	1,210	1,320
12	120	240	360	480	600	720	840	960	1,080	1,200	1,320	1,440

– Gattegno charts

$$2590 \div 10 = 259$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

$$45 \div 100 = 0.45$$

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

– place value charts

$$7900 \div 10 = 790$$

1,000s	100s	10s	1s
7	9	0	0
	7	9	0

$$45 \div 100 = 0.45$$

10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s
4	5		
	0	4	5

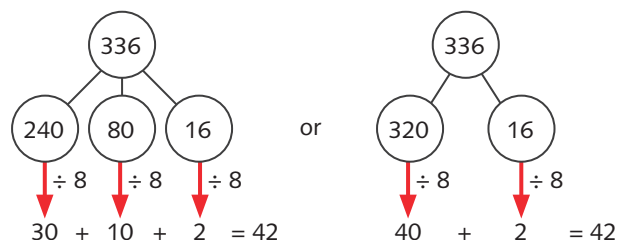
Year 4 (P5 NI)

DIVISION Continued

Mental strategies continued

- part-whole models

$$336 \div 8 = 42$$



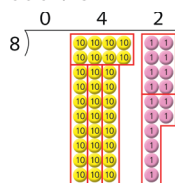
- place value counters

$$1800 \div 300 = 6$$

$$1800 \div 3 = 600$$



$$336 \div 8 = 42$$



- Make connections between arrays, number patterns and counting in steps of a constant size
- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number,

$$486 \div 9 = (450 \div 9) + (36 \div 9)$$

$$= 50 + 4$$

$$= 54$$

Written methods

- Short division (without a remainder):
 - Divide a two-digit or three-digit number by a one-digit number (TO $\div$ O/HTO $\div$ O)
- Estimate and check the answer to a calculation

Expanded written method

$$486 \div 9$$

$$\begin{array}{r}
 54 \\
 9 \overline{) 486} \\
 \underline{- 450} \quad (50 \times 9) \\
 36 \\
 \underline{- 36} \quad (4 \times 9) \\
 0
 \end{array}$$

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (50×9) and (4×9) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many nines divide into 480 so that the answer is a multiple of 10? (50) There are 50 nines or 450, with 36 remaining. How many nines in 36? (4) So 486 divided by nine is 54.'

Year 4 (P5 NI)

DIVISION Continued

Written methods continued

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of short division, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10, with subtracting whole numbers mentally, and in their understanding of place value.

Formal written method of short division

$$\begin{array}{r} 5 \ 4 \\ 9 \overline{) 4 \ 8 \ 3 \ 6} \end{array} \quad \text{or} \quad \begin{array}{r} 5 \ 4 \\ 9 \overline{) 4 \ 48 \ 3 \ 6} \end{array}$$

As there are no groups of 9 hundreds in 400, you can also write a superscript 4 to represent the 4 hundreds. It is written in front of the 8 tens to show that there are 48 tens.

The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 3 represents the 3 tens that are remaining after 9 has been divided into 480. It is written in front of the 6 to show that 36 now has to be divided by 9.

Year 5 (P6 NI)

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- find 10, 100, 1000, 10 000 or 100 000 more or less than a given number
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers with up to three decimal places
- identify the value of each digit in numbers given to three decimal places
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- add numbers mentally with increasingly large numbers
- add decimals, including a mix of one-digit whole numbers and tenths, e.g. $1.3 + 0.5$; and complements of 1, e.g. $0.83 + 0.17$
- add whole numbers with more than four digits, including using the formal written method (columnar addition)
- add decimals to two places, including using the formal written method (columnar addition)
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, regroup, combine, addition, add, plus, more, 1 more, 2 more ... 10 more, 100 more, ... 100 000 more, how many, how much, how many more to make, count on, count forwards, double, altogether, addend, augend, sum, total, larger, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, commutative, associative, decimal, decimal point

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving number up to three decimal places

Year 5 (P6 NI)

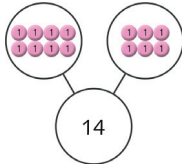
ADDITION Continued

Mental strategies

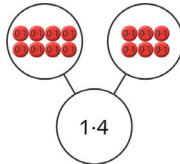
- Continue to use core representations, models and images when necessary:

- part-whole models

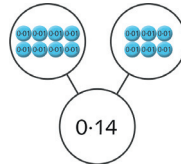
$$8 + 6 = 14$$



$$0.8 + 0.6 = 1.4$$



$$0.08 + 0.06 = 0.14$$



- tenths and hundredths addition and subtraction tables

+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.6	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
0.7	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11
0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13
0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14
0.05	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15
0.06	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16
0.07	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17
0.08	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.09	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
0.1	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2

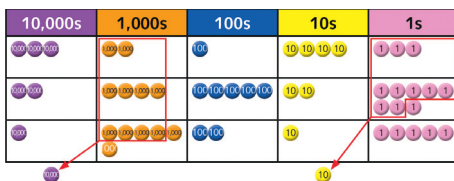
- empty number lines

$$0.47 + 0.76 = 1.23$$

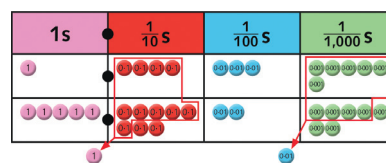


- place value counters and/or place value charts

$$32,143 + 24,528 + 16,215 = 72,886$$



$$1.436 + 5.827 = 7.263$$



- Develop further the relationship between addition and subtraction
- Continue to apply knowledge of the commutative law, e.g.
 - put the larger number first and count on in steps of 1, 10, 100 or 1000;
 - partition additions into hundreds, tens and ones, then recombine, e.g. $356 + 57 = 356 + 50 + 7$
 $= 406 + 7$
 $= 413$
- Identify near doubles, using doubles already known, e.g. $1.7 + 1.8$
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. $9 + 7 = 16$ and $0.9 + 0.7 = 1.6$
- Use knowledge of the associative law when adding more than two numbers, e.g. $24 + 27 + 16 = (24 + 16) + 27$
 $= 40 + 27$
 $= 67$

Year 5 (P6 NI)

ADDITION Continued

Written methods

- Add whole numbers with more than four digits
- Add decimals with up to two decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar addition

$$12\,957 + 14\,635$$

$$\begin{array}{r} 1\,2\,9\,5\,7 \\ + 1\,4\,6\,3\,5 \\ \hline 2\,7\,5\,9\,2 \\ \hline \end{array}$$

1 1

$$56.47 + 84.86$$

$$\begin{array}{r} 5\,6\,.\,4\,7 \\ + 8\,4\,.\,8\,6 \\ \hline 1\,4\,1\,.\,3\,3 \\ \hline \end{array}$$

1 1 1

Regrouped digits are recorded below the line, using the phrases 'regroup 10 ones into 1 ten', and/or 'regroup 10 tens into 1 hundred' and/or 'regroup 10 hundreds into 1 thousand'.

Where appropriate, place value columns are labelled, e.g. TO·th, to remind children of the value of each of the digits.

If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar addition.

Year 5 (P6 NI)

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- subtract numbers mentally with increasingly large numbers
- subtract decimals, including a mix of one-digit whole numbers and tenths, e.g. $1.8 - 0.7$
- subtract whole numbers with more than four digits, including using the formal written method (columnar subtraction)
- subtract decimals to two places, including using the formal written method (columnar subtraction)
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, exchange, take away, how many are left, how much is left, subtraction, subtract, minus, minuend, subtrahend, difference, difference between, less, 1 less, 2 less ... 10 less, 100 less, ... 100 000 less, fewer, decrease, how many more is... than..., leaves, count on, count back, count backwards, count up, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, decimal, decimal point

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

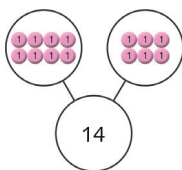
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving numbers up to three decimal places

Mental strategies

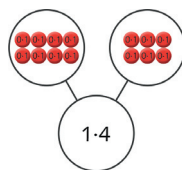
- Continue to use core representations, models and images when necessary:

- part-whole models

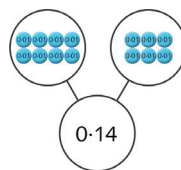
$$14 - 6 = 8$$



$$1.4 - 0.6 = 0.8$$



$$0.14 - 0.06 = 0.08$$



- tenths and hundredths addition and subtraction tables

+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.6	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
0.7	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11
0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13
0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14
0.05	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15
0.06	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16
0.07	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17
0.08	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.09	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
0.1	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2

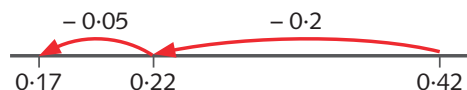
Year 5 (P6 NI)

SUBTRACTION Continued

Mental strategies continued

- empty number lines

$$0.42 - 0.25 = 0.17$$

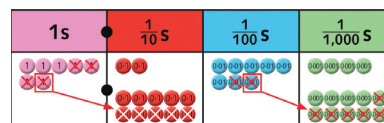


- place value counters and/or place value charts

$$40\,582 - 26\,735 = 13\,847$$



$$7.284 - 3.516 = 3.768$$



- Develop further the relationship between addition and subtraction
- Calculate mentally a difference such as $12\,462 - 2300$ by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. $16 - 9 = 7$ and $1.6 - 0.9 = 0.7$
- Use partitioning, e.g. $456 - 84 = 456 - 80 - 4$
 $= 376 - 4$
 $= 372$

Written methods

- Subtract whole numbers with more than four digits
- Subtract decimals with up to two decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition/exchanging)

$$45\,257 - 17\,488$$

$$83.72 - 36.49$$

$$\begin{array}{r} \overset{3}{4} \overset{14}{5} \overset{11}{2} \overset{14}{5} \overset{17}{7} \\ - 1\,7\,4\,8\,8 \\ \hline 2\,7\,7\,6\,9 \end{array}$$

or

$$\begin{array}{r} \overset{3}{4} \overset{14}{5} \overset{11}{2} \overset{14}{5} \overset{1}{7} \\ - 1\,7\,4\,8\,8 \\ \hline 2\,7\,7\,6\,9 \end{array}$$

You can also write the exchanged value like this.

$$\begin{array}{r} \overset{7}{8} \overset{13}{3} \overset{6}{7} \overset{12}{2} \\ - 3\,6\,4\,9 \\ \hline 4\,7\,2\,3 \end{array}$$

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens, then the hundreds, then the thousands and finally the tens of thousands. Refer to subtracting the tens, for example, by saying '14 tens subtract 8 tens', not '14 subtract 8'.

In the first example the ones, tens, hundreds and thousands to be subtracted are all larger than all of the ones, tens, hundreds and thousands you are subtracting from so 1 ten needs to be exchanged for 10 ones, 1 hundred exchanged for 10 tens, 1 thousand for 10 hundreds, and 1 ten thousand for 10 thousands.

The calculation begins by exchanging one of the 5 tens for 10 ones, crossing out the 5 and writing a superscript 4, and crossing out the 7 and writing a superscript 17. The calculation then becomes 17 subtract 8.

You then exchange one of the 2 hundreds for 10 tens, crossing out the 2 and writing a superscript 1, and writing a superscript 1 in front of the 4 to make 14 tens. The calculation then becomes 14 tens subtract 8 tens.

Next, you exchange one of the 5 thousands for 10 hundreds, crossing out the 5 and writing a superscript 4, and writing a superscript 1 in front of the 1 to make 11 hundreds. The calculation then becomes 11 hundreds subtract 4 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 4 to make 14 thousands. The calculation then becomes 14 thousands subtract 7 thousands.

Then finally 30 000 subtract 10 000.

Where appropriate, place value columns are labelled, e.g. TO-th, to remind children of the value of each of the digits.

If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar subtraction (decomposition/exchanging).

Year 5 (P6 NI)

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of the multiplication facts for multiplication tables up to 12×12
- use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, e.g. $70 \times 80 = 5600$
- continue to use place value, known and derived facts to multiply mentally
- multiply whole numbers and those involving decimals by 10, 100 and 1000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- continue to recognise commutativity in mental calculations
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, decompose, regroup, combine, how many, how much, groups of, same size, equal groups, plus, add, repeated addition, in each group, lots of, groups of, counting in steps of ..., multiplication, times, multiply, multiplied by, product, factor, multiple of, 10 times greater, 100 times greater, 1000 times greater, repeated addition, array, row, column, double, twice, altogether, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, commutative, associative, decimal, decimal point, place holder

Reason mathematically and solve problems

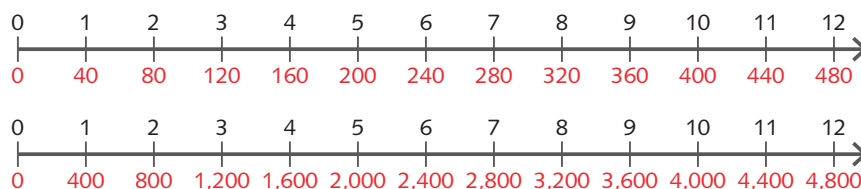
Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems, involving multiplication including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates

Mental strategies

- Continue to use core representations, models and images when necessary:

– number lines

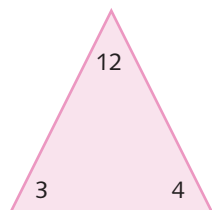


Year 5 (P6 NI)

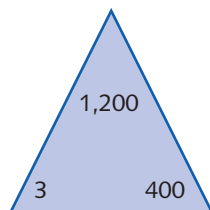
MULTIPLICATION Continued

Mental strategies continued

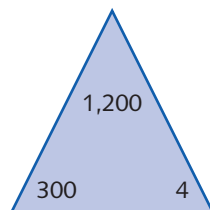
– trios



$$\begin{aligned} 3 \times 4 &= 12 \\ 4 \times 3 &= 12 \\ 12 \div 4 &= 3 \\ 12 \div 3 &= 4 \end{aligned}$$



$$\begin{aligned} 3 \times 400 &= 1,200 \\ 400 \times 3 &= 1,200 \\ 1,200 \div 400 &= 3 \\ 1,200 \div 3 &= 400 \end{aligned}$$



$$\begin{aligned} 300 \times 4 &= 1,200 \\ 4 \times 300 &= 1,200 \\ 1,200 \div 4 &= 300 \\ 1,200 \div 300 &= 4 \end{aligned}$$

– multiplication square to 12×12 / tenths and hundredths multiplication squares

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

×	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
2	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6
4	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4	4.4	4.8
5	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
6	0.6	1.2	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6.6	7.2
7	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7	7.7	8.4
8	0.8	1.6	2.4	3.2	4	4.8	5.6	6.4	7.2	8	8.8	9.6
9	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	9.9	10.8
10	1	2	3	4	5	6	7	8	9	10	11	12
11	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9	11	12.1	13.2
12	1.2	2.4	3.6	4.8	6	7.2	8.4	9.6	10.8	12	13.2	14.4

×	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
2	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2	0.22	0.24
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.3	0.33	0.36
4	0.04	0.08	0.12	0.16	0.2	0.24	0.28	0.32	0.36	0.4	0.44	0.48
5	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6
6	0.06	0.12	0.18	0.24	0.3	0.36	0.42	0.48	0.54	0.6	0.66	0.72
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.7	0.77	0.84
8	0.08	0.16	0.24	0.32	0.4	0.48	0.56	0.64	0.72	0.8	0.88	0.96
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9	0.99	1.08
10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
11	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.1	1.21	1.32
12	0.12	0.24	0.36	0.48	0.6	0.72	0.84	0.96	1.08	1.2	1.32	1.44

Year 5 (P6 NI)

MULTIPLICATION Continued

Mental strategies continued

- Gattegno charts

$$257 \times 10 = 2570$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

$$2.69 \times 100 = 269$$

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

- place value charts

$$483 \times 100 = 48\,300$$

10,000s	1,000s	100s	10s	1s
		4	8	3
4	8	3	0	0

$$27.95 \times 10 = 279.5$$

100s	10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s	$\frac{1}{1,000}$ s
	2	7	9	5	
2	7	9	5		

- place value counters

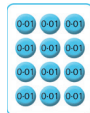
$$3 \times 4 = 12$$

$$3 \times 0.04 = 0.12$$

$$3264 \times 3 = 9792$$



100 times smaller



x	3,000	200	60	4
3	9,000	600	180	12

$$27 \times 34 = 918$$

x	20	7
30	600	210
4	80	28

- Continue to use the inverse relationship between multiplication and division
- Use related facts and doubling and halving:
 - double or halve the most significant digit first
 - to multiply by 25, multiply by 100 then divide by 4
 - double one number and halve the other
 - find the multiplication facts for the $\times 16$ multiplication table by doubling the $\times 8$ multiplication facts
- Use closely related facts:
 - multiply by 19 or 21 by multiplying by 20 and adjusting
 - develop the $\times 14$ multiplication table by adding facts from the $\times 12$ and $\times 2$ multiplication tables
- Use factors, e.g. $9 \times 18 = 9 \times 6 \times 3$
- Use patterns of similar calculations, e.g. $8 \times 6 = 48$ and $8 \times 600 = 4800$
- Understand and use the commutative law, e.g. $14 \times 12 = (2 \times 7) \times 12$

$$= 2 \times (7 \times 12)$$

$$= 2 \times 84$$

$$= 168$$
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one digit number, or two two-digit numbers, e.g.

$$\begin{aligned} 378 \times 4 &= (300 \times 4) + (70 \times 4) + (8 \times 4) \\ &= 1200 + 280 + 32 \\ &= 1512 \end{aligned}$$

$$\begin{aligned} 78 \times 34 &= (78 \times 30) + (78 \times 4) \\ &= 2340 + 312 \\ &= 2652 \end{aligned}$$

Year 5 (P6 NI)

MULTIPLICATION Continued

Written methods

- Short multiplication:
 - Multiply numbers up to four digits by a one-digit number (HTO $\times$ O/ThHTO $\times$ O)
- Estimate and check the answer to a calculation

Grid method

$$378 \times 4$$

$$\begin{array}{r} \times \quad 300 \quad 70 \quad 8 \\ 4 \quad \boxed{2100} \quad \boxed{280} \quad \boxed{32} \end{array} = 1512$$

Expanded written method

$$378 \times 4$$

$$\begin{array}{r} \quad 3 \quad 7 \quad 8 \\ \times \quad \quad 4 \\ \hline \quad 3 \quad 2 \quad (8 \times 4) \\ \quad 2 \quad 8 \quad 0 \quad (70 \times 4) \\ \quad 1 \quad 2 \quad 0 \quad 0 \quad (300 \times 4) \\ \hline 1 \quad 5 \quad 1 \quad 2 \\ \hline 1 \end{array}$$

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the partial calculations: (8×4) , (70×4) and (300×4) may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 378×4 it is 'seventy multiplied by four', not 'seven multiplied by four', although the relationship 7×4 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of short multiplication, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10, 100 and 1000, with adding whole numbers mentally, and in their understanding of place value.

Formal written method of short multiplication

$$378 \times 4$$

$$\begin{array}{r} \quad 3 \quad 7 \quad 8 \\ \times \quad 3 \quad 3 \quad 4 \\ \hline 1 \quad 5 \quad 1 \quad 2 \end{array} \quad \text{or} \quad \begin{array}{r} \quad 3 \quad 7 \quad 8 \\ \times \quad \quad 4 \\ \hline \quad 1 \quad 5 \quad 1 \quad 2 \\ \quad 3 \quad 3 \end{array}$$

You can also write the regrouped value like this.

The grid method and expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Year 5 (P6 NI)

MULTIPLICATION Continued

Written methods continued

- Long multiplication:
 - Multiply numbers up to three digits by a two-digit number (TO $\times$ TO/HTO $\times$ TO)
- Estimate and check the answer to a calculation

Grid method

$$78 \times 44$$

	70	8	
30	2100	240	
4	280	32	

$$\begin{array}{r}
 2\ 3\ 4\ 0 \\
 +\ 3\ 1\ 2 \\
 \hline
 2\ 6\ 5\ 2
 \end{array}$$

Expanded written method

$$\begin{array}{r}
 7\ 8 \\
 \times\ 3\ 4 \\
 \hline
 3\ 2\quad (8 \times 4) \\
 2\ 8\ 0\quad (70 \times 4) \\
 2\ 4\ 0\quad (8 \times 30) \\
 2\ 1\ 0\ 0\quad (70 \times 30) \\
 \hline
 2\ 6\ 5\ 2 \\
 1
 \end{array}$$

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the partial calculations: (8×4) , (70×4) , (8×30) and (70×30) may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 78×4 it is 'seventy multiplied by four', not 'seven multiplied by four', although the relationship 7×4 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of long multiplication, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with adding whole numbers mentally, and in their understanding of place value.

Formal written method

$$78 \times 34$$

$$\begin{array}{r}
 7\ 8 \\
 \times\ 3\ 4 \\
 \hline
 3\ 1\ 32\quad (78 \times 4) \\
 2\ 3\ 24\ 0\quad (78 \times 30) \\
 \hline
 2\ 6\ 5\ 2
 \end{array}$$

The grid method and expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, as indicated in the example above, the partial calculations: (78×4) and (78×30) may be recorded to the right of the algorithm. Also, again where appropriate, place value columns may be labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Year 5 (P6 NI)

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of the division facts for multiplication tables up to 12×12
- use known division facts to derive related facts involving multiples of 10, 100 and 1000, e.g. $6300 \div 90 = 70$
- continue to use place value, known and derived facts to divide mentally
- divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, decompose, regroup, exchange, how many, how much, groups of, share, equal groups of, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into, division, divide, divided between, divided equally between, divided evenly between, divided by, divided into, dividend, divisor, quotient, 10 times smaller, 100 times smaller, 1000 times smaller, left, left over, remainder, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, decimal, decimal point, place holder

Reason mathematically and solve problems

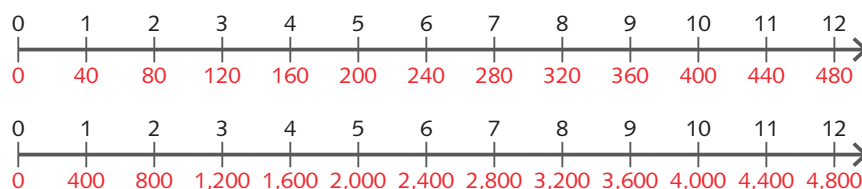
Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates

Mental strategies

- Continue to use core representations, models and images when necessary:

– number lines

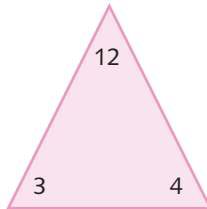


Year 5 (P6 NI)

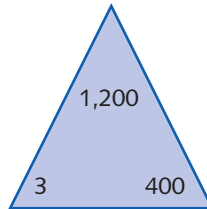
DIVISION Continued

Mental strategies continued

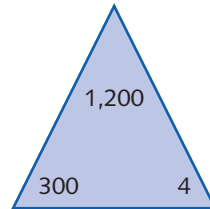
– trios



$$\begin{aligned} 3 \times 4 &= 12 \\ 4 \times 3 &= 12 \\ 12 \div 4 &= 3 \\ 12 \div 3 &= 4 \end{aligned}$$



$$\begin{aligned} 3 \times 400 &= 1,200 \\ 400 \times 3 &= 1,200 \\ 1,200 \div 400 &= 3 \\ 1,200 \div 3 &= 400 \end{aligned}$$



$$\begin{aligned} 300 \times 4 &= 1,200 \\ 4 \times 300 &= 1,200 \\ 1,200 \div 4 &= 300 \\ 1,200 \div 300 &= 4 \end{aligned}$$

– multiplication square to 12 x 12 / tenths and hundredths multiplication squares

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
2	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6
4	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4	4.4	4.8
5	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
6	0.6	1.2	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6.6	7.2
7	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7	7.7	8.4
8	0.8	1.6	2.4	3.2	4	4.8	5.6	6.4	7.2	8	8.8	9.6
9	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	9.9	10.8
10	1	2	3	4	5	6	7	8	9	10	11	12
11	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9	11	12.1	13.2
12	1.2	2.4	3.6	4.8	6	7.2	8.4	9.6	10.8	12	13.2	14.4

x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
2	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2	0.22	0.24
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.3	0.33	0.36
4	0.04	0.08	0.12	0.16	0.2	0.24	0.28	0.32	0.36	0.4	0.44	0.48
5	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6
6	0.06	0.12	0.18	0.24	0.3	0.36	0.42	0.48	0.54	0.6	0.66	0.72
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.7	0.77	0.84
8	0.08	0.16	0.24	0.32	0.4	0.48	0.56	0.64	0.72	0.8	0.88	0.96
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9	0.99	1.08
10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
11	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.1	1.21	1.32
12	0.12	0.24	0.36	0.48	0.6	0.72	0.84	0.96	1.08	1.2	1.32	1.44

Year 5 (P6 NI)

DIVISION Continued

Mental strategies continued

- Gattegno charts

$$3680 \div 10 = 368$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

$$368 \div 1000 = 0.368$$

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

- place value charts

$$12\,680 \div 10 = 1268$$

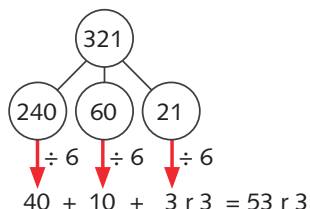
10,000s	1,000s	100s	10s	1s
1	2	6	8	0
	1	2	6	8

$$1\,596 \div 1000 = 1.596$$

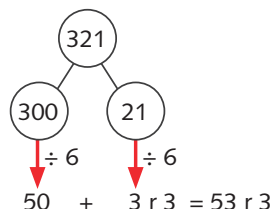
1,000s	100s	10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s	$\frac{1}{1,000}$ s
1	5	9	6			
		1	5	9	6	

- part-whole models

$$321 \div 6 = 53 \text{ r } 3$$



or

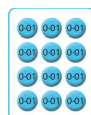


- place value counters

$$12 \div 4 = 3$$

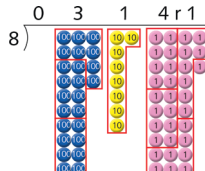


100 times smaller



$$0.12 \div 0.04 = 3$$

$$2513 \div 8 = 314 \text{ r } 1$$



- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number,

$$486 \div 9 = (450 \div 9) + (36 \div 9)$$

$$= 50 + 4$$

$$= 54$$

Year 5 (P6 NI)

DIVISION Continued

Written methods

- Short division (including with remainders expressed as a whole number, fraction or decimal):
 - Divide numbers up to 4 digits by a one-digit number ($\text{HTO} \div \text{O}$ / $\text{ThHTO} \div \text{O}$)
- Estimate and check the answer to a calculation

Expanded written method

$$279 \div 6$$

$$\begin{array}{r} 46 \text{ r } 3 \\ 6 \overline{) 279} \\ \underline{- 240} \quad (40 \times 6) \\ 39 \\ \underline{- 36} \quad (6 \times 6) \\ 3 \end{array}$$

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (40×6) and (3×6) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many sixes divide into 270 so that the answer is a multiple of 10?' (40) There are 40 sixes or 240, with 39 remaining.

Children then ask: 'How many sixes in 39?' (6, with 3 remaining) So 279 divided by six is 46 remainder 3. Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of short division, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10, with subtracting whole numbers mentally, and in their understanding of place value.

Formal written method of short division

Whole number remainder

$$279 \div 6$$

$$6 \overline{) 279} \text{ r } 3 \quad \text{or} \quad 6 \overline{) 2^{27}9} \text{ r } 3$$

As there are no groups of 6 hundreds in 200, you can also write a superscript 2 to represent the 2 hundreds. It is written in front of the 7 tens to show that there are 27 tens.

Fraction remainder

$$279 \div 6$$

$$6 \overline{) 279} \frac{1}{2}$$

Decimal remainder

$$279 \div 6$$

$$6 \overline{) 279.0} 46.5$$

The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Children should describe what they are doing using phrases similar to the following: 'How many sixes divide into 270 so that the answer is a multiple of 10?' (40) There are 40 sixes or 240, with 30 remaining. The superscript 3 represents the 3 tens that are remaining after 6 has been divided into 270. It is written in front of the 9 to show that 39 now has to be divided by 6.

Children then ask: 'How many sixes in 39?' (6 remainder 3). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

Year 6 (P7 NI)

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- find 10, 100, 1000, 10 000 or 100 000 more or less than a given number
- round any whole number to a required degree of accuracy

DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers with up to three decimal places
- identify the value of each digit in numbers given to three decimal places
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- practise addition for larger numbers and decimals, using the formal written method of columnar addition
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To add successfully, pupils need to understand and use the following key words and phrases:

millions, hundreds of thousands, tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, regroup, combine, addition, add, plus, more, 1 more, 2 more ... 10 more, 100 more, ... 1 000 000 more, how many, how much, how many more to make, count on, count forwards, double, altogether, addend, augend, sum, total, larger, increase, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, properties of arithmetic, commutative, associative, compensation strategies, decimal, decimal point

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Year 6 (P7 NI)

ADDITION Continued

Mental strategies

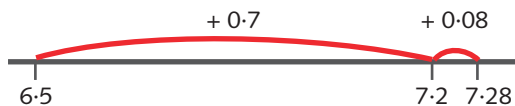
- Continue to use core representations, models and images when necessary:
 - tenths and hundredths addition and subtraction tables

+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.6	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
0.7	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11
0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13
0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14
0.05	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15
0.06	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16
0.07	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17
0.08	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.09	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
0.1	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2

- empty number lines

$$6.5 + 0.78 = 7.28$$

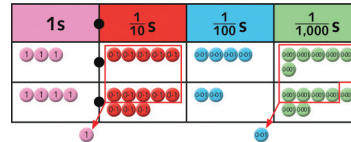


- place value counters and/or place value charts

$$275\,316 + 150\,837 = 426\,153$$



$$3.546 + 4.827 = 8.373$$



- Develop further the relationship between addition and subtraction
- Continue to apply knowledge of the commutative law, e.g.
 - put the larger number first and count on in steps of 1, 10, 100 or 1000
 - partition additions into hundreds, tens and ones, then recombine, e.g. $3356 + 257 = 3356 + 200 + 50 + 7 = 3556 + 57 = 3613$
- Identify near doubles, using doubles already known, e.g. $5.7 + 5.8$
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. $9 + 7 = 16$ and $0.09 + 0.07 = 0.16$
- Use knowledge of the associative law when adding more than two numbers, e.g. $24 + 27 + 16 = (24 + 16) + 27 = 40 + 27 = 67$

Year 6 (P7 NI)

ADDITION Continued

Written methods

- Add numbers with more than four digits
- Add decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar addition

$$456\,287 + 359\,849$$

$$57\cdot486 + 45\cdot378$$

$$\begin{array}{r} 4\,5\,6\,2\,8\,7 \\ +\,3\,5\,9\,8\,4\,9 \\ \hline 8\,1\,6\,1\,3\,6 \\ \hline 1\,1\,1\,1\,1 \end{array}$$

$$\begin{array}{r} 5\,7\cdot4\,8\,6 \\ +\,4\,5\cdot3\,7\,8 \\ \hline 1\,0\,2\cdot8\,6\,4 \\ \hline 1\,1\,1 \end{array}$$

Regrouped digits are recorded below the line, using phrases 'regroup 10 ones into 1 ten', and/or 'regroup 10 tens into 1 hundred' and/or 'regroup 10 hundreds into 1 thousand' and/or 'regroup 10 thousands into 1 tens of thousand'.

Where appropriate, place value columns are labelled, e.g. TO·thth, to remind children of the value of each of the digits.

If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar addition.

Year 6 (P7 NI)

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- practise subtraction for larger numbers and decimals, using the formal written method of columnar subtraction
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To subtract successfully, pupils need to understand and use the following key words and phrases:

millions, hundreds of thousands, tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, exchange, take away, how many are left, how much is left, subtraction, subtract, minus, minuend, subtrahend, difference, difference between, less, 1 less, 2 less ... 10 less, 100 less, ... 1 000 000 less, fewer, decrease, how many more is... than..., leaves, count on, count back, count backwards, count up, is the same as, is equal to, equal, equals, number sentence, calculation, calculate, related facts, fact family, opposite operations, inverse operations, inverse relationship, decimal, decimal point

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Mental strategies

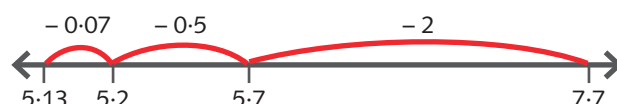
- Continue to use core representations, models and images when necessary:
 - tenths and hundredths addition and subtraction tables

+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.6	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
0.7	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11
0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13
0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14
0.05	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15
0.06	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16
0.07	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17
0.08	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.09	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
0.1	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2

- empty number lines

$$7.7 - 2.57 = 5.13$$



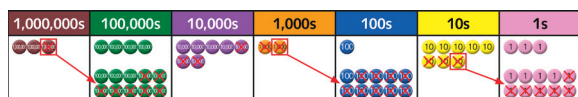
Year 6 (P7 NI)

SUBTRACTION Continued

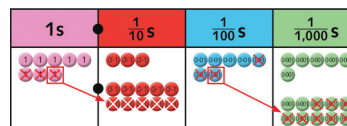
Mental strategies continued

- place value counters and/or place value charts

$$3\,472\,183 - 731\,926 = 2\,740\,257$$



$$8.376 - 2.528 = 5.848$$



- Develop further the relationship between addition and subtraction
- Calculate mentally a difference such as $23\,004 - 18\,998$ by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. $16 - 9 = 7$ and $0.16 - 0.09 = 0.07$
- Use partitioning, e.g. $4656 - 358 = 4656 - 300 - 50 - 8$
 $= 4356 - 58$
 $= 4298$

Written methods

- Subtract numbers with more than four digits
- Subtract decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition/exchanging)

$$746\,291 - 298\,354$$

$$\begin{array}{r} \overset{6}{7} \overset{13}{4} \overset{15}{6} \overset{12}{2} \overset{8}{9} \overset{11}{1} \\ - 2\,9\,8\,3\,5\,4 \\ \hline 4\,4\,7\,9\,3\,7 \end{array}$$

or

$$\begin{array}{r} \overset{6}{7} \overset{13}{4} \overset{15}{6} \overset{12}{2} \overset{8}{9} \overset{1}{1} \\ - 2\,9\,8\,3\,5\,4 \\ \hline 4\,4\,7\,9\,3\,7 \end{array}$$

You can also write the exchanged values like this.

$$63.237 - 45.869$$

$$\begin{array}{r} \overset{5}{6} \overset{12}{3} \overset{11}{2} \overset{12}{3} \overset{17}{7} \\ - 4\,5\,8\,6\,9 \\ \hline 1\,7\,3\,6\,8 \end{array}$$

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens ... and finally the hundreds of thousands. Refer to subtracting the tens, for example, by saying '8 tens subtract 5 tens', not '8 subtract 5'.

In the first example, the ones, hundreds, thousands and tens of thousands to be subtracted are all larger than all of the ones, hundreds, thousands and tens of thousands you are subtracting from.

The calculation begins by exchanging one of the 9 tens for 10 ones, crossing out the 9 and writing a superscript 8, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 subtract 4.

You then calculate 8 tens subtract 5 tens.

Next, you exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 hundreds subtract 3 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 5 to make 15 thousands. The calculation then becomes 15 thousands subtract 8 thousands.

Next, you exchange one of the 7 hundreds of thousands for 10 tens of thousands, crossing out the 7 and writing a superscript 6, and writing a superscript 1 in front of the 3 to make 13 tens of thousands. The calculation then becomes 13 tens of thousands subtract 9 tens of thousands.

Then, finally, 600 000 subtract 200 000.

Where appropriate, place value columns are labelled, e.g. TO-thth, to remind children of the value of each of the digits.

If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar subtraction (decomposition/exchanging).

Year 6 (P7 NI)

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of the multiplication facts for multiplication tables up to 12×12
- use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g. $70 \times 80 = 5600$, $0.8 \times 6 = 4.8$
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- continue to multiply whole numbers and those involving decimals by 10, 100 and 1000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- continue to recognise commutativity in mental calculations
- practise multiplication for larger numbers, using the formal written method of short multiplication
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- multiply one-digit numbers with up to two decimal places by whole numbers
- multiply numbers with up to two decimal places by one- and two-digit whole numbers
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To multiply successfully, pupils need to understand and use the following key words and phrases:

millions, hundreds of thousands, tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, decompose, regroup, combine, how many, how much, groups of, same size, equal groups, plus, add, repeated addition, in each group, lots of, groups of, counting in steps of ..., multiplication, times, multiply, multiplied by, product, factor, multiple of, 10 times greater, 100 times greater, 1000 times greater, increase, decrease, repeated addition, array, row, column, double, twice, altogether, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, commutative, associative, distributive, compensation strategies, decimal, decimal point, place holder

Reason mathematically and solve problems

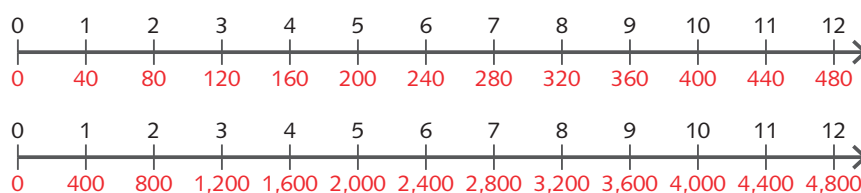
Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Mental strategies

- Continue to use core representations, models and images when necessary:

– number lines

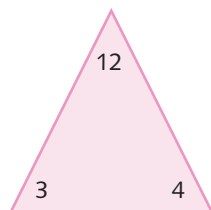


Year 6 (P7 NI)

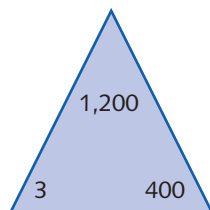
MULTIPLICATION Continued

Mental strategies continued

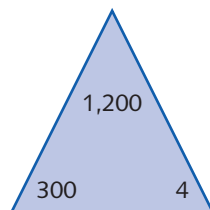
– trios



$$\begin{aligned} 3 \times 4 &= 12 \\ 4 \times 3 &= 12 \\ 12 \div 4 &= 3 \\ 12 \div 3 &= 4 \end{aligned}$$



$$\begin{aligned} 3 \times 400 &= 1,200 \\ 400 \times 3 &= 1,200 \\ 1,200 \div 400 &= 3 \\ 1,200 \div 3 &= 400 \end{aligned}$$



$$\begin{aligned} 300 \times 4 &= 1,200 \\ 4 \times 300 &= 1,200 \\ 1,200 \div 4 &= 300 \\ 1,200 \div 300 &= 4 \end{aligned}$$

– multiplication square to 12 x 12 / tenths and hundredths multiplication squares

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

×	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
2	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6
4	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4	4.4	4.8
5	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
6	0.6	1.2	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6.6	7.2
7	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7	7.7	8.4
8	0.8	1.6	2.4	3.2	4	4.8	5.6	6.4	7.2	8	8.8	9.6
9	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	9.9	10.8
10	1	2	3	4	5	6	7	8	9	10	11	12
11	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9	11	12.1	13.2
12	1.2	2.4	3.6	4.8	6	7.2	8.4	9.6	10.8	12	13.2	14.4

×	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
2	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2	0.22	0.24
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.3	0.33	0.36
4	0.04	0.08	0.12	0.16	0.2	0.24	0.28	0.32	0.36	0.4	0.44	0.48
5	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6
6	0.06	0.12	0.18	0.24	0.3	0.36	0.42	0.48	0.54	0.6	0.66	0.72
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.7	0.77	0.84
8	0.08	0.16	0.24	0.32	0.4	0.48	0.56	0.64	0.72	0.8	0.88	0.96
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9	0.99	1.08
10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
11	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.1	1.21	1.32
12	0.12	0.24	0.36	0.48	0.6	0.72	0.84	0.96	1.08	1.2	1.32	1.44

Year 6 (P7 NI)

MULTIPLICATION Continued

Mental strategies continued

- Gattegno charts

$$24.69 \times 10 = 246.9$$

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

$$1.46 \times 1000 = 1460$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

- place value charts

$$5438 \times 100 = 543\,800$$

100,000s	10,000s	1,000s	100s	10s	1s
5	4	3	8	0	0

$$165.439 \times 10 = 1654.39$$

1,000s	100s	10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s	$\frac{1}{1,000}$ s
1	6	5	4	3	9	

- place value counters

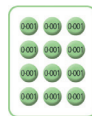
$$3 \times 4 = 12$$

$$3 \times 0.004 = 0.012$$

$$2654 \times 3 = 7962$$



1,000 times smaller



$\times$	2,000	600	50	4
3	6,000	1,800	150	12

$$27 \times 34 = 918$$

$\times$	20	7
30	600	210
4	80	28

- Continue to use the inverse relationship between multiplication and division
- Use related facts and doubling and halving:
 - double or halve the most significant digit first
 - to multiply by 25, multiply by 100 then divide by 4
 - double one number and halve the other
 - find the multiplication facts for the $\times 24$ multiplication table by doubling the $\times 12$ multiplication facts
 - squares of multiples of 10 to 100, e.g. 70×70 , and the corresponding halves
 - doubles of decimals, e.g. 4.7×2 , 0.63×2 , and the corresponding halves
 - doubles of multiples of 10 to 1000, e.g. 830×2 , and the corresponding halves
 - doubles of multiples of 100 to 10 000, e.g. $48\,500 \times 2$, and the corresponding halves
- Use closely related facts:
 - multiply by 49 or 51 by multiplying by 50 and adjusting
 - develop the $\times 18$ multiplication table by adding facts from the $\times 10$ and $\times 8$ multiplication tables
- Use factors, e.g. $9 \times 18 = 9 \times 6 \times 3$
- Use patterns of similar calculations, e.g. $8 \times 6 = 48$ and $0.8 \times 6 = 4.8$
- Continue to use and apply the commutative law
- Understand and use the associative law, e.g. $10.6 \times 30 = 10.6 \times (10 \times 3)$
or $(10.6 \times 10) \times 3$
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, or two two-digit numbers, and partitioning when multiplying a whole number or decimal by a one-digit number, e.g.

$$\begin{aligned} 285 \times 63 &= (200 \times 63) + (80 \times 63) + (5 \times 63) \\ &= 12\,600 + 5040 + 315 \\ &= 17\,955 \end{aligned}$$

$$\begin{aligned} 4.83 \times 6 &= (4 \times 6) + (0.8 \times 6) + (0.03 \times 6) \\ &= 24 + 4.8 + 0.18 \\ &= 28.98 \end{aligned}$$

Year 6 (P7 NI)

MULTIPLICATION Continued

Written methods

- Short multiplication (whole numbers):
 - Multiply multi-digit numbers up to 4 digits by a one-digit whole number
- Estimate and check the answer to a calculation

Formal written method of short multiplication

$$5643 \times 8$$

$$\begin{array}{r} 5643 \\ \times 8 \\ \hline 45144 \end{array} \quad \text{or} \quad \begin{array}{r} 5643 \\ \times 8 \\ \hline 45144 \\ \hline 532 \end{array}$$

You can also write the regrouped values like this.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

If necessary, remind children of the grid method and/or expanded written method (see below) so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of short multiplication.

- Short multiplication (Decimals):
 - Multiply one-digit or two-digit numbers with up to two decimal places by a one-digit number
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Grid method

$$7.56 \times 8$$

$$\begin{array}{|c|c|c|} \hline \times & 7 & 0.5 & 0.06 \\ \hline 8 & 56 & 4 & 0.48 \\ \hline \end{array} = 60.48$$

Expanded written method

$$7.56 \times 8$$

$$\begin{array}{r} 7.56 \\ \times 8 \\ \hline 0.48 \quad (0.06 \times 8) \\ 4.00 \quad (0.5 \times 8) \\ 56.00 \quad (7 \times 8) \\ \hline 60.48 \\ 1 \end{array}$$

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the partial calculations: (0.06×8) , (0.5×8) and (7×8) may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g. when multiplying the tenths in 7.56×8 it is 'zero point five multiplied by eight', not 'five multiplied by eight'. Although the relationship 5×8 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. TO.th, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of short multiplication, will depend on how secure the children are in their recall of the multiplication and division facts, including involving decimals with up to two decimal places, with adding decimals mentally, and in their understanding of place value.

Formal written method of short multiplication

$$\begin{array}{r} 7.56 \\ \times 8 \\ \hline 60.48 \end{array} \quad \text{or} \quad \begin{array}{r} 7.56 \\ \times 8 \\ \hline 60.48 \\ \hline 4 \quad 4 \end{array}$$

You can also write the regrouped values like this.

The grid method and the expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, place value columns are labelled, e.g. TO.th, to remind children of the value of each of the digits.

Year 6 (P7 NI)

MULTIPLICATION Continued

Written methods continued

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Grid method

$$7.56 \times 8$$

7.56×8 is equivalent to $756 \times 8 \div 100$

$\times$	700	50	6	
8	5600	400	48	$= 6048$

$$6048 \div 100 = 6.48$$

Expanded written method

7.56×8 is equivalent to $756 \times 8 \div 100$

	7	5	6	
$\times$			8	
		4	8	(6×8)
	4	0	0	(50×8)
5	6	0	0	(700×8)
6	0	4	8	
	1			

$$6048 \div 100 = 6.48$$

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the partial calculations: (6×8) , (50×8) and (700×8) may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g. when multiplying the tens in 756×8 it is 'fifty multiplied by eight', not 'five multiplied by eight'. Although the relationship 5×8 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of short multiplication, will depend on how secure the children are in their recall of the multiplication and division facts, including involving multiples of 10, 100 and 1000, with adding whole numbers mentally, and in their understanding of place value.

Formal written method of short multiplication

7.56×8 is equivalent to $756 \times 8 \div 100$

$\begin{array}{r} 756 \\ \times 8 \\ \hline 6048 \end{array}$	or	$\begin{array}{r} 756 \\ \times 8 \\ \hline 6048 \\ \hline 44 \end{array}$	You can also write the regrouped values like this.
---	----	--	---

$$6048 \div 100 = 6.48$$

The grid method and the expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Year 6 (P7 NI)

MULTIPLICATION Continued

Written methods continued

- Long multiplication (whole numbers):
 - Multiply multi-digit numbers up to four digits by a two-digit number (TO $\times$ TO / HTO $\times$ TO)
- Estimate and check the answer to a calculation

Grid method

$$285 \times 63$$

$\times$	200	80	5	
60	12 000	4800	300	1 7 1 0 0
3	600	240	15	+ 8 5 5
				<u>1 7 9 5 5</u>

Expanded written method

$$\begin{array}{r}
 285 \\
 \times 63 \\
 \hline
 15 \quad (5 \times 3) \\
 240 \quad (80 \times 3) \\
 600 \quad (200 \times 3) \\
 300 \quad (5 \times 60) \\
 4800 \quad (80 \times 60) \\
 12000 \quad (200 \times 60) \\
 \hline
 17955
 \end{array}$$

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the six partial calculations may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g. when multiplying the tens in 285×3 it is 'eighty multiplied by three', not 'eight multiplied by three'. Although the relationship 8×3 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. TThThHTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of long multiplication will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with adding whole numbers mentally, and in their understanding of place value.

Formal written method of long multiplication

$$285 \times 63$$

$$\begin{array}{r}
 285 \\
 \times 63 \\
 \hline
 825 \quad (285 \times 3) \\
 1710 \quad (285 \times 60) \\
 \hline
 17955
 \end{array}$$

The grid method and the expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, as indicated in the example above, the partial calculations: (285×3) and (285×60) may be recorded to the right of the algorithm. Also again where appropriate, place value columns are labelled, e.g. TThThHTO, to remind children of the value of each of the digits.

Year 6 (P7 NI)

MULTIPLICATION Continued

Written methods continued

- Long multiplication (Decimals):
 - Multiply one-digit numbers with up to two decimal places by a two-digit number
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Grid method

$$5.68 \times 76 =$$

×	5	0.6	0.08	
70	350	42	5.6	
6	30	3.6	0.48	

	3	9	7	·	6	0
+		3	4	·	0	8
	4	3	1	·	6	8
	1		1			

Expanded written method

	5	·	6	8		
×		7	6			
	0	·	4	8	(0.08 × 6)	
	3	·	6	0	(0.6 × 6)	
	3	0	·	0	0 (5 × 6)	
	5	·	6	0	(0.08 × 70)	
	4	2	·	0	0 (0.6 × 70)	
	3	5	0	·	0	0 (5 × 70)
	4	3	1	·	6	8
	1		1			

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the six partial calculations may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g. when multiplying the tenths in 5.68×6 it is 'zero point six multiplied by six', not 'six multiplied by six'. Although the relationship 6×6 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO.th, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of long multiplication, will depend on how secure the children are in their recall of the multiplication and division facts, including involving multiples of 10 and decimals with up to two decimal places, with adding decimal numbers mentally, and in their understanding of place value.

Formal written method of long multiplication

	5	·	6	8				
×		7	6					
	3	4	·	0	8 (5.68 × 6)			
	3	9	4	7	5	·	6	0 (5.68 × 70)
	4	3	1	·	6	8		
	1		1					

The grid method and the expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, as indicated in the example above, the partial calculations: (5.68×6) and (5.68×70) may be recorded to the right of the algorithm. Also, again where appropriate, place value columns may be labelled, e.g. HTO.th, to remind children of the value of each of the digits.

Year 6 (P7 NI)

MULTIPLICATION Continued

Written methods continued

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Grid method

5.68×76 is equivalent to $568 \times 76 \div 100$

$\times$	500	60	8	
70	35 000	4200	560	3 9 7 6 0
6	3 000	360	48	+ 3 4 0 8
				4 3 1 6 8
				1 1

$43,168 \div 100 = 432.68$

Expanded written method

5.68×76 is equivalent to $568 \times 76 \div 100$

	5	6	8	
$\times$		7	6	
		4	8	(8 × 6)
		3	6	0 (60 × 6)
	3	0	0	0 (500 × 6)
		5	6	0 (8 × 70)
		4	2	0 0 (60 × 70)
	3	5	0	0 0 (500 × 70)
	4	3	1	6 8
	1	1	1	

$43,168 \div 100 = 432.68$

The first step is to show all of the calculations involved. For the expanded written method, where appropriate, as indicated in the example above, the six partial calculations may be recorded to the right of the algorithm.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g. when multiplying the tens in 568×6 it is 'sixty multiplied by six', not 'six multiplied by six'. Although the relationship 6×6 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. TThThHTO, to remind children of the value of each of the digits.

The amount of time that should be spent teaching and practising the grid method and/or the expanded written method, before progressing onto the formal written method of long multiplication, will depend on how secure the children are in their recall of the multiplication and division facts, including involving multiples of 10 and 100, with adding whole numbers mentally, and in their understanding of place value.

Formal written method of long multiplication

5.68×76 is equivalent to $568 \times 76 \div 100$

	5	6	8	
$\times$		7	6	
	3	4 ⁴	0 ⁴	8 (568 × 6)
	3	9 ⁴	7 ⁵	6 0 (568 × 70)
	4	3	1	6 8
	1	1		

$43,168 \div 100 = 432.68$

The grid method and the expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Where appropriate, as indicated in the example above, the partial calculations: (568×6) and (568×70) may be recorded to the right of the algorithm. Also, again where appropriate, place value columns may be labelled, e.g. TThThHTO, to remind children of the value of each of the digits.

Year 6 (P7 NI)

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of the division facts for multiplication tables up to 12×12
- use known division facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g. $6300 \div 90 = 70$, $6.3 \div 9 = 0.7$
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- continue to divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- practise division for larger numbers, using the formal written method of short division
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- divide numbers with up to two decimal places by one- and two-digit whole numbers
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Key language

To divide successfully, pupils need to understand and use the following key words and phrases:

millions, hundreds of thousands, tens of thousands, thousands, hundreds, tens, ones, tenths, hundredths, thousandths, partition, decompose, regroup, exchange, how many, how much, groups of, share, equal groups of, equal sharing, unequal sharing, equal parts, unequal parts, share equally between, share into, division, divide, divided between, divided equally between, divided evenly between, divided by, divided into, dividend, divisor, quotient, 10 times smaller, 100 times smaller, 1000 times smaller, increase, decrease, left, left over, remainder, is the same as, is equal to, equal, equals, number sentence, opposite operations, inverse operations, inverse relationship, compensation strategies decimal, decimal point, place holder, divisibility

Reason mathematically and solve problems

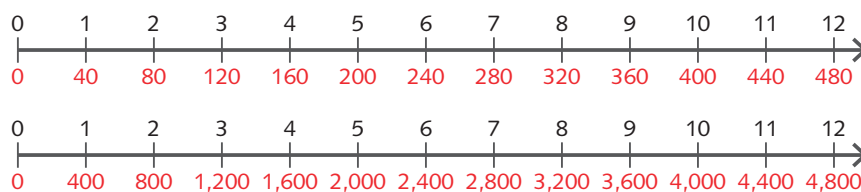
Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Mental strategies

- Continue to use core representations, models and images when necessary:

– number lines

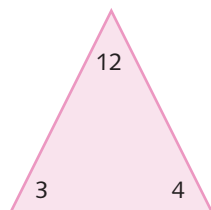


Year 6 (P7 NI)

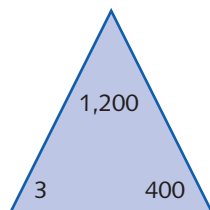
DIVISION Continued

Mental strategies continued

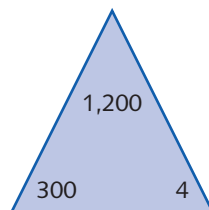
– trios



$$\begin{aligned} 3 \times 4 &= 12 \\ 4 \times 3 &= 12 \\ 12 \div 4 &= 3 \\ 12 \div 3 &= 4 \end{aligned}$$



$$\begin{aligned} 3 \times 400 &= 1,200 \\ 400 \times 3 &= 1,200 \\ 1,200 \div 400 &= 3 \\ 1,200 \div 3 &= 400 \end{aligned}$$



$$\begin{aligned} 300 \times 4 &= 1,200 \\ 4 \times 300 &= 1,200 \\ 1,200 \div 4 &= 300 \\ 1,200 \div 300 &= 4 \end{aligned}$$

– multiplication square to 12 x 12 / tenths and hundredths multiplication squares

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

×	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
2	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6
4	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4	4.4	4.8
5	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
6	0.6	1.2	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6.6	7.2
7	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7	7.7	8.4
8	0.8	1.6	2.4	3.2	4	4.8	5.6	6.4	7.2	8	8.8	9.6
9	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	9.9	10.8
10	1	2	3	4	5	6	7	8	9	10	11	12
11	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9	11	12.1	13.2
12	1.2	2.4	3.6	4.8	6	7.2	8.4	9.6	10.8	12	13.2	14.4

×	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
2	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2	0.22	0.24
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.3	0.33	0.36
4	0.04	0.08	0.12	0.16	0.2	0.24	0.28	0.32	0.36	0.4	0.44	0.48
5	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6
6	0.06	0.12	0.18	0.24	0.3	0.36	0.42	0.48	0.54	0.6	0.66	0.72
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.7	0.77	0.84
8	0.08	0.16	0.24	0.32	0.4	0.48	0.56	0.64	0.72	0.8	0.88	0.96
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9	0.99	1.08
10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
11	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.1	1.21	1.32
12	0.12	0.24	0.36	0.48	0.6	0.72	0.84	0.96	1.08	1.2	1.32	1.44

Year 6 (P7 NI)

DIVISION Continued

Mental strategies continued

- Gattegno charts

$$24.69 \div 10 = 2.469$$

10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

$$357.8 \div 100 = 3.578$$

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

- place value charts

$$631 \div 100 = 6.31$$

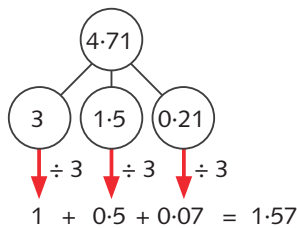
100s	10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s
6	3	1		
			6	3
				1

$$341.68 \div 10 = 34.168$$

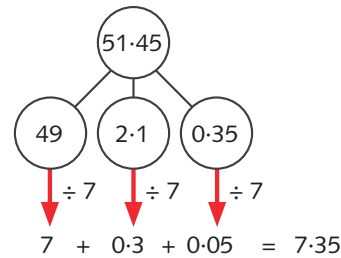
100s	10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s	$\frac{1}{1,000}$ s
3	4	1	6	8	
			3	4	1
				6	8

- part-whole models

$$4.71 \div 3 = 1.57$$



$$51.45 \div 7 = 7.35$$



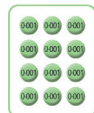
- place value counters

$$12 \div 4 = 3$$

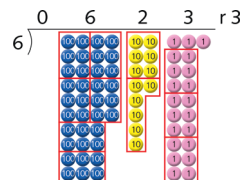


1,000 times smaller

$$0.012 \div 0.004 = 3$$



$$3741 \div 6 = 623.5$$



- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number, $486 \div 9 = (450 \div 9) + (36 \div 9)$

$$= 50 + 4$$

$$= 54$$

Year 6 (P7 NI)

DIVISION Continued

Written methods

- Short division, including with remainders expressed as a whole number, fraction or decimal (whole numbers)
 - Divide numbers up to 4 digits by a one-digit number (HTO ÷ O/ThHTO ÷ O)
- Estimate and check the answer to a calculation

Formal written method of short division

Whole number remainder

$$1838 \div 8$$

$$\begin{array}{r} 229 \text{ r } 6 \\ 8 \overline{) 1838} \\ \underline{8} \\ 0 \\ \underline{16} \\ 2 \\ \underline{16} \\ 0 \end{array}$$

Fraction remainder

$$1838 \div 8$$

$$\begin{array}{r} 229 \frac{3}{4} \\ 8 \overline{) 1838} \\ \underline{8} \\ 0 \\ \underline{16} \\ 2 \\ \underline{16} \\ 0 \end{array}$$

Decimal remainder

$$1838 \div 8$$

$$\begin{array}{r} 229.75 \\ 8 \overline{) 1838.00} \\ \underline{8} \\ 0 \\ \underline{16} \\ 2 \\ \underline{16} \\ 0 \\ \underline{16} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

Children should describe what they are doing using phrases similar to the following: 'How many eights divide into 1800 so that the answer is a multiple of 100?' (200) There are 200 eights or 1600, with 200 remaining. The superscript 2 represents the 2 hundreds that are remaining after 8 has been divided into 1800. It is written in front of the 3 to show that a total of 23 tens (230) now have to be divided by eight.

Children then ask: 'How many eights divide into 230 so that the answer is a multiple of 10?' (20) There are 20 eights or 160, with 70 remaining. The superscript 7 represents the 7 tens that are remaining after 8 has been divided into 230. It is written in front of the 8 to show that 78 now has to be divided by 8.

Children then ask: 'How many eights in 78?' (9 remainder 6). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

If necessary, remind children of the expanded written method (see below) so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of short division.

- Short division (Decimals)
 - Divide numbers with up to two decimal places by a one-digit number (O-th ÷ O/TO-th ÷ O)
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Expanded written method

$$51.45 \div 7$$

$$\begin{array}{r} 7 \cdot 3 \cdot 5 \\ 7 \overline{) 51.45} \\ \underline{49} \\ 02 \\ \underline{02} \\ 00 \\ \underline{00} \\ 00 \\ \underline{00} \\ 00 \end{array}$$

(7 × 7) (0.3 × 7) (0.05 × 7)

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (7 × 7) (0.3 × 7) and (0.05 × 7) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many sevens divide into 51.45?' (7) There are 7 sevens with 2.45 remaining. Most children should be able to work out 51.45 subtract 49 mentally.

Children then ask: 'How many sevens divide into 2.45?' (0.3) There are 3 tenths with 35 hundredths remaining.

Children then ask: 'How many sevens in 0.35?' (0.05)

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of short division, will depend on how secure the children are in their recall of the multiplication and division facts, including involving decimals with up to two decimal places, with subtracting decimal numbers mentally, and in their understanding of place value.

Year 6 (P7 NI)

DIVISION Continued

Written methods continued

Formal written method of short division

$$\begin{array}{r} 7 \overline{) 51.2435} \end{array} \quad \text{or} \quad \begin{array}{r} 7 \overline{) 5^5 1.2435} \end{array}$$

As there are no groups of 7 tens in 50, you can also write a superscript 5 to represent the 5 tens. It is written in front of the 1 to show that there are 51 ones.

The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Children should describe what they are doing using phrases similar to the following: 'How many sevens divide into 51?' (7) There are 7 sevens with 2 ones remaining. The superscript 2 represents the 2 ones that are remaining after 7 has been divided into 51. It is written in front of the 4 tenths to show that 24 tenths now have to be divided by 7.

Children then ask: 'How many sevens in 24 tenths?' (3) There are 3 tenths with 3 tenths remaining. The superscript 3 represents the 3 tenths that are remaining after 7 has been divided into 24 tenths. It is written in front of the 5 hundredths to show that 35 hundredths now have to be divided into 7.

Children then ask: 'How many sevens in 35 hundredths?' (5 hundredths).

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method

$51.45 \div 7$ is equivalent to $5145 \div 7 \div 100$

$$\begin{array}{r} 7 \overline{) 5145} \\ - 4900 \quad (700 \times 7) \\ \hline 245 \\ - 210 \quad (30 \times 7) \\ \hline 35 \\ - 35 \quad (5 \times 7) \\ \hline 00 \end{array}$$

$$735 \div 100 = 7.35$$

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (700×7) , (30×7) and (5×7) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many sevens divide into 5145 so that the answer is a multiple of 100?' (700) There are 700 sevens or 4900, with 245 remaining. Most children should be able to work out 5145 subtract 4900 mentally.

Children then ask: 'How many sevens divide into 245 so that the answer is a multiple of 10?' (30) There are 30 sevens or 210, with 35 remaining.

Children then ask: 'How many sevens in 35?' (5).

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of short division, will depend on how secure the children are in their recall of the multiplication and division facts, including involving multiples of 10 and 100, with subtracting whole numbers mentally, and in their understanding of place value.

Year 6 (P7 NI)

DIVISION Continued

Written methods continued

Formal written method of short division

$51.45 \div 7$ is equivalent to $5145 \div 7 \div 100$

$$\begin{array}{r} 7 \ 3 \ 5 \\ 7 \overline{) 5 \ 1 \ 4 \ 5} \end{array}$$

$$735 \div 100 = 7.35$$

The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Phrases similar to those on page 86 for the formal written method of short division involving whole numbers should be used.

- Long division, including with remainders expressed as a whole number, fraction or decimal (Whole numbers)
 - Divide numbers up to 4 digits by a two-digit number (HTO $\div$ TO/ThHTO $\div$ TO)
- Estimate and check the answer to a calculation

Expanded written method

$$\begin{array}{r} 5 \ 3 \ 4 \ r \ 4 \\ 16 \overline{) 8 \ 5 \ 4 \ 8} \\ - \ 8 \ 0 \ 0 \ 0 \quad (500 \times 16) \\ \hline 4 \ 5 \ 4 \ 8 \\ - \ 4 \ 8 \ 0 \quad (39 \times 16) \\ \hline 6 \ 8 \\ - \ 6 \ 4 \quad (4 \times 16) \\ \hline 4 \end{array}$$

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (500×16) , (39×16) and (4×16) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many 16s divide into 8548 so that the answer is a multiple of 100?' (500) There are 500 groups of 16 or 8000, with 548 remaining.

Children then ask: 'How many 16s divide into 548 so that the answer is a multiple of 10?' (30) There are 30 groups of 16 or 480, with 68 remaining. Most children should be able to subtract 480 from 548 mentally.

Children then ask: 'How many 16s in 68?' (4 with 4 remaining). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of long division, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting whole numbers mentally, and in their understanding of place value.

Formal written method of long division

$$\begin{array}{r} 5 \ 3 \ 4 \ r \ 4 \\ 16 \overline{) 8 \ 5 \ 4 \ 8} \\ - \ 8 \ 0 \quad \downarrow \\ \hline 4 \ 5 \ 4 \quad \downarrow \\ - \ 4 \ 8 \quad \downarrow \\ \hline 6 \ 8 \\ - \ 6 \ 4 \\ \hline 4 \end{array}$$

The expanded written method leads to the formal written method of long division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Year 6 (P7 NI)

DIVISION Continued

Written methods continued

Children should describe what they are doing using phrases similar to the following: 'How many 16s divide into 85 hundreds?' (5 hundreds and a remainder) 16 groups of 5 hundreds equals 80 hundreds. Subtract 80 hundreds from 85 hundreds to find the remainder (5 hundreds). Exchange the remaining 5 hundreds for 50 tens and combine with the existing 4 tens to make 54 tens.

Children then ask: 'How many 16s in 54 tens?' (3 tens and a remainder). 16 groups of 3 tens equals 48 tens. Subtract 48 tens from 54 tens to find the remainder (6 tens). Most children should be able to subtract 48 tens from 54 tens mentally. Exchange the remaining 6 tens for 60 ones and combine with the existing 8 ones to make 68 ones.

Children then ask: 'How many 16s in 68?' (4 and a remainder). 16 groups of 4 equals 64. Subtract 64 from 68 to find the remainder (4). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

- Long division (Decimals)
 - Divide numbers with up to two decimal places by a two-digit whole number ($TO\text{-}th \div TO$)
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Expanded written method

$$45.64 \div 14$$

$$\begin{array}{r}
 3 \cdot 2 \cdot 6 \\
 14 \overline{) 45 \cdot 64} \\
 - 42 \cdot 00 \quad (3 \times 14) \\
 \hline
 03 \cdot 64 \\
 - 02 \cdot 80 \quad (0.2 \times 14) \\
 \hline
 00 \cdot 84 \\
 - 00 \cdot 84 \quad (0.06 \times 14) \\
 \hline
 00 \cdot 00
 \end{array}$$

The first step is to show all of the calculations involved. Where appropriate, as indicated in the example above, the partial calculations: (3×14) , (0.2×14) and (0.06×14) may be recorded to the right of the algorithm.

Children should describe what they are doing using phrases similar to the following: 'How many 14s divide into 45.64?' (3) There are 3 groups of 14 or 42, with 3.64 remaining.

Children then ask: 'How many 14s divide into 3.64?' (0.2) There are 2 tenths, with 84 hundredths remaining. Most children should be able to subtract 2.8 from 3.64 mentally.

Children then ask: 'How many 14s in 0.84?' (0.06).

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of long division, will depend on how secure the children are in their recall of multiplication and division facts, including involving decimals with up to two decimal places, with subtracting decimals mentally, and in their understanding of place value.

Year 6 (P7 NI)

DIVISION Continued

Written methods continued

Formal written method of long division

$$\begin{array}{r}
 3 \cdot 2 \cdot 6 \\
 14 \overline{) 45 \cdot 64} \\
 \underline{- 42 \cdot} \\
 36 \\
 \underline{- 28} \\
 084 \\
 \underline{- 084} \\
 000
 \end{array}$$

The expanded written method leads to the formal written method of long division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Children should describe what they are doing using phrases similar to the following: 'How many 14s divide into 45?' (3 and a remainder) 14 groups of 3 equals 42. Subtract 42 from 45 to find the remainder (3). Combine with the existing 6 tenths to make 3·6.

Children then ask: 'How many 14s in 3·6?' (2 tenths and a remainder). 14 groups of 2 tenths equals 2·8. Subtract 2·8 from 3·6 to find the remainder (0·8). Most children should be able to subtract 2·8 from 3·6 mentally. Combine with the existing 4 hundredths to make 0·84.

Children then ask: 'How many 14s in 0·84?' (0·06).

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method

$45\cdot64 \div 14$ is equivalent to $4564 \div 14 \div 100$

$$\begin{array}{r}
 3 \quad 2 \quad 6 \\
 14 \overline{) 4564} \\
 \underline{- 4200} \quad (300 \times 14) \\
 364 \\
 \underline{- 280} \quad (20 \times 14) \\
 84 \\
 \underline{- 84} \quad (6 \times 14) \\
 00
 \end{array}$$

$$326 \div 100 = 3\cdot26$$

Phrases similar to those on page 88 for the expanded written method of long division involving whole numbers should be used.

The amount of time that should be spent teaching and practising the expanded written method, before progressing onto the formal written method of long division, will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting whole numbers mentally, and in their understanding of place value.

Year 6 (P7 NI)

DIVISION Continued

Written methods continued

Formal written method of long division

$45.64 \div 14$ is equivalent to $4564 \div 14 \div 100$

$$\begin{array}{r}
 326 \\
 14 \overline{) 4564} \\
 \underline{- 42} \\
 36 \\
 \underline{- 28} \\
 84 \\
 \underline{- 84} \\
 00
 \end{array}$$

$$326 \div 100 = 3.26$$

The expanded written method leads to the formal written method of long division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Phrases similar to those on pages 88 and 89 for the formal written method of long division involving whole numbers should be used.